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Study of Statistical Variations
of Load Spectra and Material Properties
on Aircraft Fatigue Life

by

Richard W. Walter II

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Thesis Advisor: Gerald H. Lindsey

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Study of Statistical Variations of Load Spectra and Material Properties on Aircraft Fatigue Life

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Submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

NAVAIR utilizes the fatigue spectrum of an existing Navy aircraft to set the structural design requirements for a new Navy aircraft. The current design requirement is for the new aircraft to withstand a fatigue spectrum at least as severe as the spectrum experienced by 99.73% (3 standard deviations) of the aircraft from which the design requirement originated. Two years of A-6 data were used in the study, which contained the number of g exceedences at the four g, five g, six g, and seven g levels.

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TABLE OF CONTENTS

I. INTRODUCTION

The Navy utilizes the fatigue spectrum from a current Navy aircraft to set the structural design requirements for a new aircraft. They will base the structural design requirements for the upcoming AX, for instance, on the fatigue spectrum history of the A-6, or a similar attack aircraft. The current Navy procedure is to plot the number of four g, five g, six g, and seven g exceedences for the entire fleet of an existing, similar mission aircraft. The design for the follow-on aircraft would be required to withstand the fatigue spectrum experienced by the 99.7th percentile aircraft at each of the four exceedence levels. Since the number of exceedences for each respective g level is likely to be associated with different specific aircraft, the combined spectrum of the maximum will be more severe than the spectrum experienced by any individual aircraft in the population.

Requiring a new aircraft design to withstand a fatigue spectrum of 99.73 percent (arbitrarily based on 3σ Normal distribution) at each exceedence level for a similar operational aircraft is representative of a requirement set by NAVAIR. When setting such a requirement, it is desirable to calculate the risks involved in reducing the design requirement to a lower σ level of the population. Reducing the design requirement for the structure of the aircraft would result in decreased weight and increased performance. The range of σ levels studied for that purpose in this thesis included 3.0, 2.5, 2.0, 1.5, and 1.15.

The g exceedence database utilized for the analysis consisted of two years of A-6 counting accelerometer data. The number of exceedences per 1000 hours of flight time for the four exceedence levels were plotted on a probability plot. Several continuous distributions were utilized in an attempt to find a model that would realistically represent the number of exceedences that could be expected for an aircraft during 1000 hours of flight time. Discovery of an acceptable fit for the data provided a method of analyzing the effect on fatigue life by varying the percentile values of the probability density function.

Sensitivity studies were also carried out to test the effect on the fatigue life when varying the aircraft's reference stress. The reference stress condition was defined as the stress placed on station 165 of the wing section, for the A-6 during a symmetric, 6.5g maneuver. The reference stress was varied from 35 ksi to 100 ksi.

Additionally, numerous cycle by cycle load sequences were produced at each of the sigma levels of interest. Fatigue calculations were completed for each of the load sequences to measure the sensitivity of the fatigue life to re-ordering the load sequences.

Finally, the material properties of aluminum 7075-T651 were varied analytically to study the effect on the fatigue life of the statistical variabilities in the fatigue strength of the standard test specimen. An array of calculations were completed for different load spectrum percentiles while varying the material properties of aluminum.

Appendix A is a summary from the literature of how fatigue spectra and load sequences are generated.

II. FLIGHT MANEUVER DATA

A. THE AIRCRAFT POPULATION

The population of aircraft in the database consisted of 351 Navy and Marine A-6 aircraft. The aircraft were assigned to various reporting activities, and included aircraft from all Navy and Marine squadrons, VX-5, Naval Depots, Grumman Aircraft, Boeing Aircraft, NWC China Lake, NWL China Lake, NAV China Lake, PMTC, NWC Fallon, NV, and NSWC.

During the past few years many of the A-6's in the fleet have approached the end of their presently specified fatigue life. Since the aircraft are expected to remain in the fleet for another 20 years, steps have been taken to extend the fatigue life of the aircraft. The Navy has replaced many of the aircraft's wings or other fatigue critical components during the last few years. Approximately 120 A-6 aircraft are remaining to receive new wings. Additionally, flight restrictions have been imposed on the aircraft in order to extend the aircraft's fatigue life. Any A-6 aircraft that has an fatigue life expended (FLE) of 0.67 has consumed 67% of it's fatigue life, and is administratively restricted to a maximum of three g's or four g's of acceleration during flight The three g restriction is imposed when 67% of the fatigue life of the wing center panel has been expended, and the four g restriction becomes effective when 67% of the outer wing panel has been expended. [Ref. 1]

The database was made up of a mixture of restricted and unrestricted aircraft. Additional techniques to manage the fatigue life of the aircraft, including swapping aircraft between different squadrons to ensure that deployed squadrons had unrestricted aircraft were employed. It was expected that an unrestricted aircraft would have experienced a more severe fatigue spectrum than a restricted aircraft. Additionally, one would expect that the unrestricted fleet of aircraft would have spent the majority of their use in deployed squadrons, experiencing a much different spectrum than the mixture of restricted and unrestricted aircraft assigned to activities operating from US. based shore facilities.

B. TYPES OF DATA AND HOW MEASURED

Data utilized in the analysis was obtained from Aerostructures, INC. which is a contractor for NAVAIR [Ref. 2]. The data file contained g exceedence information for all Navy and Marine A-6 aircraft during the period December, 1989 to December, 1991. The file listed the aircraft's bureau number, flight hours, number of arrested landings, number of bolter landings, number of touch and go landings, number of field landings, number of catapults, and numbers of g exceedences at the four, five, six, and seven g levels.

The data in the file listed the data on a month by month basis. A total of 351 different bureau numbers were present in the database and 5624 lines of data were available, indicating 5624 monthly entries. Not all the aircraft had twenty four monthly entries for the two year period due to some reporting facilities reporting

data incorrectly [Ref. 1]. The 351 aircraft in the database flew a total of 186893.4 hours. The total number of g exceedences at each g level and the number of g exceedences per 1000 flight hours for the complete database are listed in Table 1.

TABLE 1: AVERAGE G EXCEEDENCES PER 1000 FLIGHT HOURS

G Exceedence Level	Total # All Data	# per 1000 Flight Hours
4g	126409.0	676.4
5g	19364.0	103.6
6g	1523.0	8.1
7g	174.0	0.931

The flight hours and the number of g exceedences that the aircraft experienced were the items of interest in the database. The g exceedences were recorded by the aircraft's counting accelerometer system. The counting accelerometer system on the A-6 registers the number of 4g, 5g, 6g, and 7g exceedences that the aircraft experiences in normal flight. The counter has four registers, one for each level of 4g, 5g, 6g, and 7g exceedences experienced during each flight. The number of exceedences that occur in a flight are recorded after each flight, and the organizational level activity is required to report the number of exceedences to the tracking organization on a monthly basis. The counting accelerometer sensor is located near the center of gravity on the A-6, and no corrections are made in the exceedence database to account for the sensor not being precisely at the aircraft's center of gravity.

The manner in which the g counting accelerometer system functions in the A-6 requires a correction to be made to the database

to obtain the true number of g exceedence readings. Since a five g maneuver by the aircraft trips both the four g and the five g counter, and a six g maneuver trips the four, five, and six g counter, etc., the higher g readings must be subtracted off as per the example in Table 2.

TABLE 2: ILLUSTRATION OF G COUNTER SYSTEM CORRECTION

	4 G's	**5 G's**	**6 G's**	**7 G's**
Actual Flight Data	10	4	2	2
Seven g Correction	None Required			
Six g Correction	=(6 g Reading)-(7 g Reading)=0			
Five g Correction	=(5 g Reading)-(6 g Corrected Reading)-(7g Reading)=2			
Four g Correction	=(4 g Reading)-(Corrected 5 g Reading)-(Corrected 6 g Reading)-(7 g Reading)=6			
Actual g Exceedences	6	2	0	2

There exist flight maneuvers for which the g counter corrections cannot be assessed. For example, a cyclic g maneuver that starts at six g's, drops down to four g's and then goes back up to six g's only trips the 4g, 5g, and 6g counter one time. The counting accelerometer has a built in reset at 2.5 g's, and will not start counting lower or intermediate g levels until the acceleration on the aircraft drops below 2.5 g's. The counting accelerometer system has no method of recording any aircraft parameters such as aircraft gross weight, speed, center of gravity location, etc. at the time of the maneuver. The counting accelerometer system also has no way of recording the sequence in which the g exceedences took place.

C. DATA PROCESSING

The data was reduced to four different populations to prepare for the statistical analysis of the data. The first step was to discard the 332 of the 5624 monthly entries of aircraft data, which had no flight hours recorded.

The second step in the data processing was to eliminate all aircraft attached to civilian support and depot level maintenance facilities. This was done because it was felt that these aircraft would not be flown in a manner similar to an attack aircraft in the fleet. This left 5116 monthly entries in the database of which all were assigned to one of the following activities: Navy or Marine squadrons, VX-5, NWC China Lake, NWL China Lake, NAV China Lake, PMTC, NWC Fallon NV, and NSWC.

The third step performed in data processing was to correct the g exceedence levels as described in the previous section. This was followed by the standardization of the data to the number of 4g, 5g, 6g, and 7g exceedences per 1000 flight hours. Standardization of the data to a common reference was required to account for different aircraft flying a different number of hours. An aircraft experiences forty 4g exceedences during twenty hours of flying is different than an aircraft experiencing forty 4g exceedences during fifty hours of flight time. In the early stages of analysis the data was standardized to per 100 flight hours and some analysis was performed on data in the per 100 flight hour format. During the intermediate and final stages of analysis all data was standardized to per 1000 flight hours

to be in agreement with most data in the literature. An example of the standardization process is displayed in Figure 1.

An example aircraft flew 35 hours in one month and the corrected four g exceedence value was 20. The number of 4 g exceedences per 1000 hours is:

$$\frac{20(exceedences)*1000(hrs)}{35(hours)} = 571\frac{exceedences}{1000\,flthrs}$$

Figure 1: Example of Data Standardization to Exceedences per 1000 Flight Hours

The fifth step in data processing was to replace all zero g exceedence values with a value of 0.0001. This was necessary since many of the statistical schemes utilized were based on the logarithm of the data. Replacing the zero data values with 0.0001 was considered to have negligible impact on the statistical analysis, since the order of magnitude between the value of 0.0001 and the minimal g exceedence level of one per 1000 flight hours is 10000.

Finally, a normalized data file was created for analysis. The curve fitting routines of the Beta function required that the random variable be restricted to a value between zero and one.

D. FINAL FORMS OF PROCESSED DATA

The first population analyzed consisted of the remaining 5116 lines of month by month listings from the database created above. This population was called the "Month by Month Population". This population was further reduced into three other databases for study.

The second population analyzed was created by summing all the g exceedences listings with the same bureau number together. For every bureau number there were 20-24 monthly listings. These were summed into one listing for each bureau number. This created a data file with the total g exceedences for the 351 aircraft in the population. This population was called the "Individual Aircraft Population".

The third population set created several populations broken out by squadron. The different bureau numbered aircraft were sorted by squadron, and then the data reduction processes listed above took place within each squadron. Month by month bureau number listings were not summed until the data was sorted to the reporting squadron, since some aircraft resided in more than one squadron during the two year period. This population was called the "Squadron Aircraft Population".

The final population was the set of all unrestricted aircraft in the database for the two year period. A SAFE report was utilized to identify those aircraft that were not on restriction during the time period of the database [Ref. 1]. One hundred three aircraft were identified as unrestricted aircraft. This population was called the "Unrestricted Aircraft Population".

III. STATISTICAL TREATMENT OF ACCELERATION DATA

A. THE RANDOM VARIABLES

There were four random variables of interest. They were the number of 4g exceedences per 1000 flight hours, the number of 5g exceedences per 1000 flight hours, the number of 6g exceedences per 1000 flight hours, and the number of 7g exceedences per 1000 flight hours. The object of the statistical analysis was to find a function that accurately models the distribution of 4g, 5g, 6g, and 7g exceedences per 1000 flight hours for the A-6 aircraft. While it is easy to calculate the average number of exceedences per 1000 flight hours for the four g levels, this provides no information as to how many exceedences an aircraft that falls in the extreme tail of the distribution might have.

The inability to identify or quantify all the variables that effect how any particular aircraft was flown over a period of 1000 flight hours precluded the selection of a model based on the underlying physics of the problem. Thus we choose the statistical approach of curve fitting the large data base of aircraft exceedences to predict the model. The data base was considered representative of what would be expected during the A-6's lifetime, since it contained data on all the A-6 aircraft over a two year recording period.

The recording of 4g, 5g, 6g, and 7g exceedences by the aircraft's g counting accelerometer system could best be described by the extreme value distribution fo the maximums. The aircraft is

experiencing some parent distribution of positive and negative g cycles, while the g recording system is recording only the maximum extreme values at the 4g, 5g, 6g, and 7g levels. Thus an attempt was made to identify a function that would describe the extreme value distribution from the 5624 line database of g exceedences. The 4g, 5g, 6g, and 7g exceedence levels were assumed to be independent and were treated as four independent distributions.

Additionally, the data was presented so that the number of g exceedences for each exceedence level could be identified for a particular aircraft at various probabilities of occurrence. In other words, if there were 100 A-6 aircraft flying 1000 hours each, and they were ranked from the least number of 4g exceedences to the largest number of 4g exceedences, then the 75th percentile aircraft, which would be the 75th aircraft, would be expected to have a specified number of 4g exceedences for the 1000 hours of flight time. This method provides an estimation of the number of g exceedences per 1000 flight hours for any aircraft distributed over the number of exceedences per 1000 flight hours curve.

An example of the desired data representation is shown on the probability plot in Figure 2. The interpretation of the probability plot is important but confusing, and is therefore explained here in detail. As shown by the arrows in Figure 2, a 75th percentile aircraft would be expected to have approximately 15 four g exceedences per

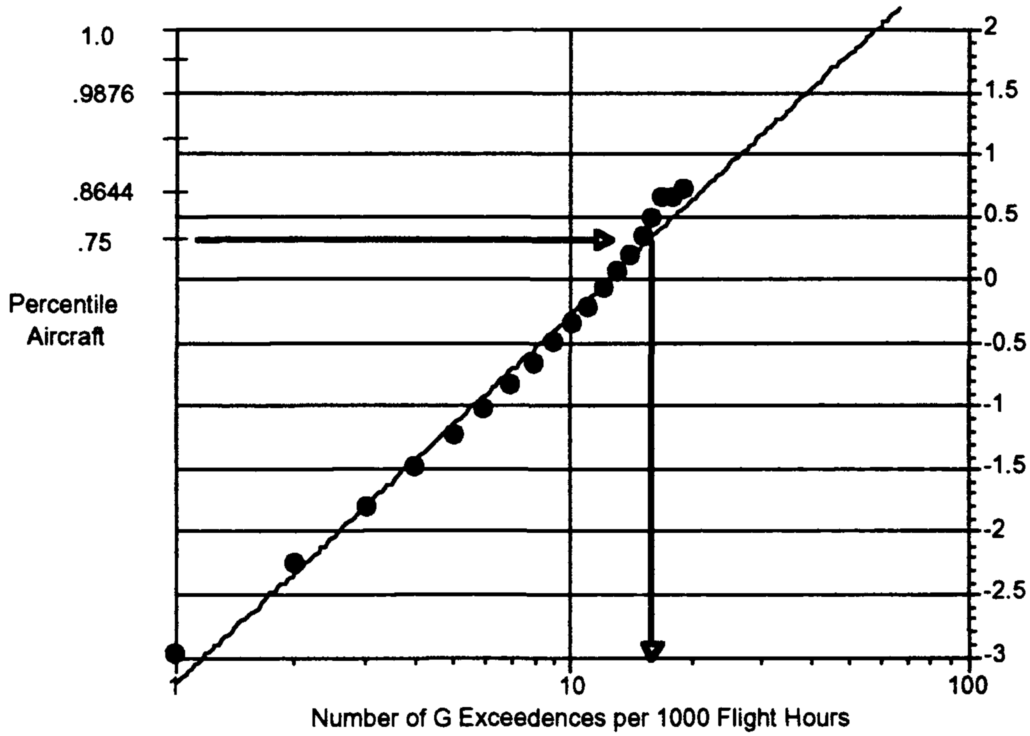

Figure 2: Data Representation by a Probability Plot

1000 flight hours. An equivalent statement would be that at least 75% of the aircraft in the database have 15, or less, four g exceedences per 1000 flight hours. Yet another correct statement would be that 25% (1.0-0.75) of the aircraft have at least 15 four g exceedences per 1000 flight hours. For the purposes of this study, we will use the statement that the 75th percentile aircraft would be expected to have experienced 15, 4 g exceedences per 1000 flight hours. If the population of aircraft at the 4g exceedence level were ranked from the least number of exceedences to the most number of exceedences, the 75th percentile aircraft would be the aircraft that fell 75% of the way through the population. Similar probability plots are desired for the 5g, 6g, and 7g exceedence levels.

B. AGSS

A statistical analysis computer program called AGSS, located on the Naval Postgraduate school mainframe computer, was utilized for the analysis of the data. The distributions that were investigated as suitable models of the data were the normal distribution, two parameter log-normal distribution, two parameter Weibull distribution, two parameter gamma distribution, and the two parameter beta distribution. The computer program utilized Maximum Likelihood Estimation routines for computation of the shape and location parameters for each of the distributions. AGSS graphically displays the probability density function (PDF), the cumulative distribution function (CDF), and the probability plot. Both axes of the probability plot were transformed to the linearized form of the CDF. Goodness of Fit indicators calculated by the computer program include the Cramer-von Mises, Anderson-Darling, and Kolmogorov-Smirnov statistics. These statistics are know as empirical distribution function (EDF) statistics and provide an overall measure of the difference between the data set and the model being tested. A significance level of at least 0.15 was required to indicate an acceptable level of fit. Further explanation of EDF statistics can be found in Chapter Four of Stevens. [Ref. 3]

The four sub-databases fitted were explained in detail in chapter II and are listed here for review:

•Full data set using month-by-month line listings.

•Summing all data of same aircraft bureau number together.

•Individual squadron data with aircraft bureau number data.

•Unrestricted aircraft data set with aircraft bureau number data.

C. RESULTS OF DISTRIBUTION FITTING

1. Population 1: Month by Month Data Population

As described in chapter II, the first database contained all fleet aircraft and aircraft assigned to the FRS, VX squadrons, PMTC, NWC, and NATC activities. All the mentioned distributions were utilized in an attempt to fit 4g, 5g, 6g, and 7g exceedence data. No acceptable curve fits were obtained. A large portion of the data at each exceedence level were zero values, complicating the possibility of a good fit. The percentage of zero values in the database was 22.4% of the 4g exceedences, 39.8% of the 5g exceedences, 84.2% of the 6g exceedences, and 98.2% of the 7g exceedences. The zero values were then discarded at each of the g exceedence levels and a curve fit was performed on the remaining data. Again no acceptable results were obtained.

2. Population 2: Individual Aircraft Population

Population two summed the monthly g exceedences with the same bureau number together. The total number of aircraft in population two was 351. Summing the monthly g exceedences together reduced the number of zero g exceedences in the population. Additionally, since the interest was in finding the number of exceedences that an aircraft would experience during 1000 flight hours of flight time, it seemed most correct to look at data summed by aircraft bureau number rather than to look at data

created by the same bureau number broken down on a month-by-month basis. All attempts to fit the population via the above mentioned distributions proved unsuccessful.

3. Population 3: Squadron Aircraft Population

Population three resulted in twenty nine data sets, one data set for each squadron. This resulted in the number of data points in each squadron being relatively small. Squadrons analyzed had as few as seven aircraft data points (VX-5) and as many as forty three (VA-42). The gamma distribution's EDF statistics indicated an acceptable fit at the 4g and 6g exceedence levels for some of the squadrons. No distribution yielded an acceptable fit at the 5g or 7g levels. Squadron data with a lower number of data points in the population often yielded acceptable fits, but the EDF statistics cannot be considered accurate for small numbers of data points and such a small sample from the original sample would not be representative of the whole population [Ref. 4]. Population three was rejected as an acceptable population for analysis due to the combination of not being able to achieve any acceptable fits at the 5g and 7g levels and the difficulties justifying a model of the population based on a few acceptable fits in such a small portion of the squadrons.

4. Population 4: Unrestricted Aircraft Population

The final population consisted of aircraft that were not restricted at any time during the time period of the database. If the A-6 database were to be utilized to set the design requirements for a follow-on aircraft; then the unrestricted aircraft database would be

the most logical choice of population for which to base the design requirements. The most extensive statistical analysis was performed on this unrestricted database.

Acceptable curve fits were obtained at the 4g level by the log-normal (Figure 3) and gamma distributions (Figure 4) and at the 5g level by the gamma distribution (Figure 5). No acceptable fits were obtained at the 6g or 7g levels. While the gamma function approximation of the 6g exceedence level data is graphically acceptable (Figure 6), the EDF statistics indicate a bad fit.

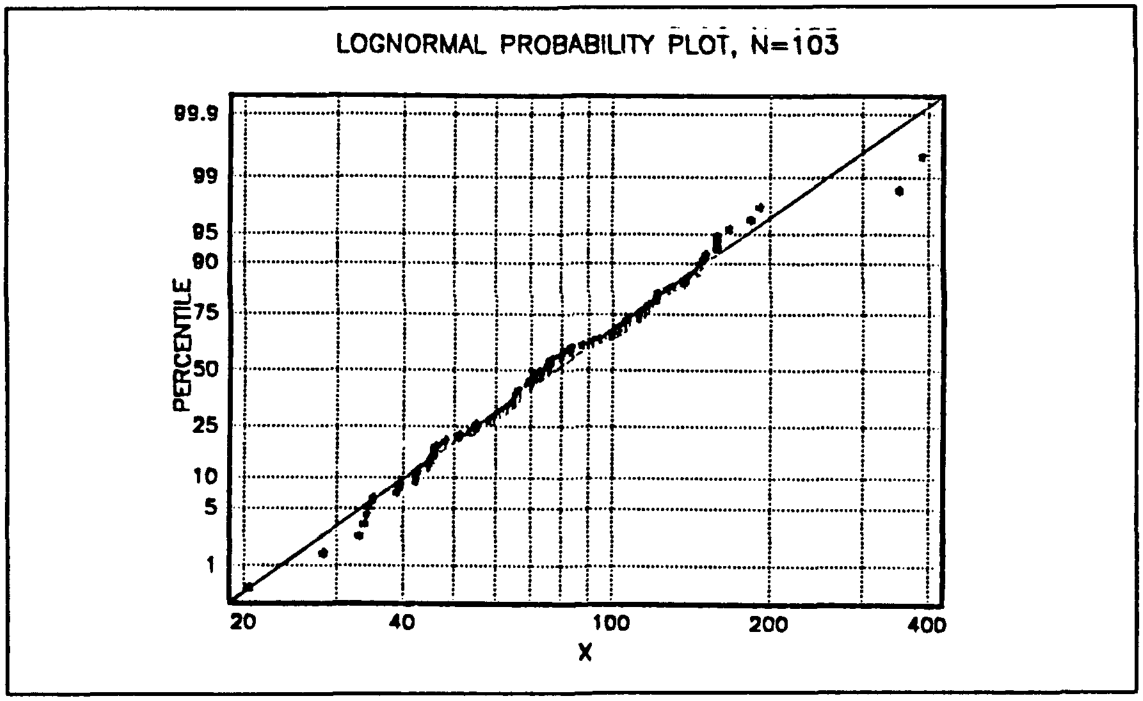

Figure 3: Log-normal Curve Fit of 4g Exceedence Level

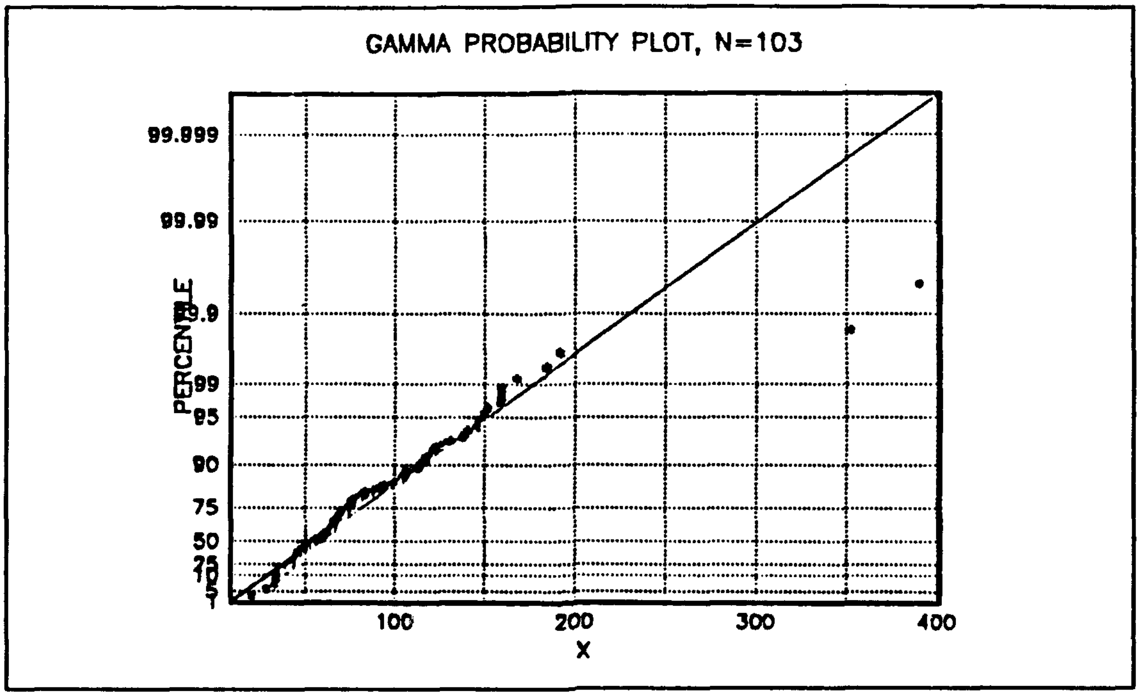

Figure 4: Gamma Curve Fit of 4g Exceedence Level

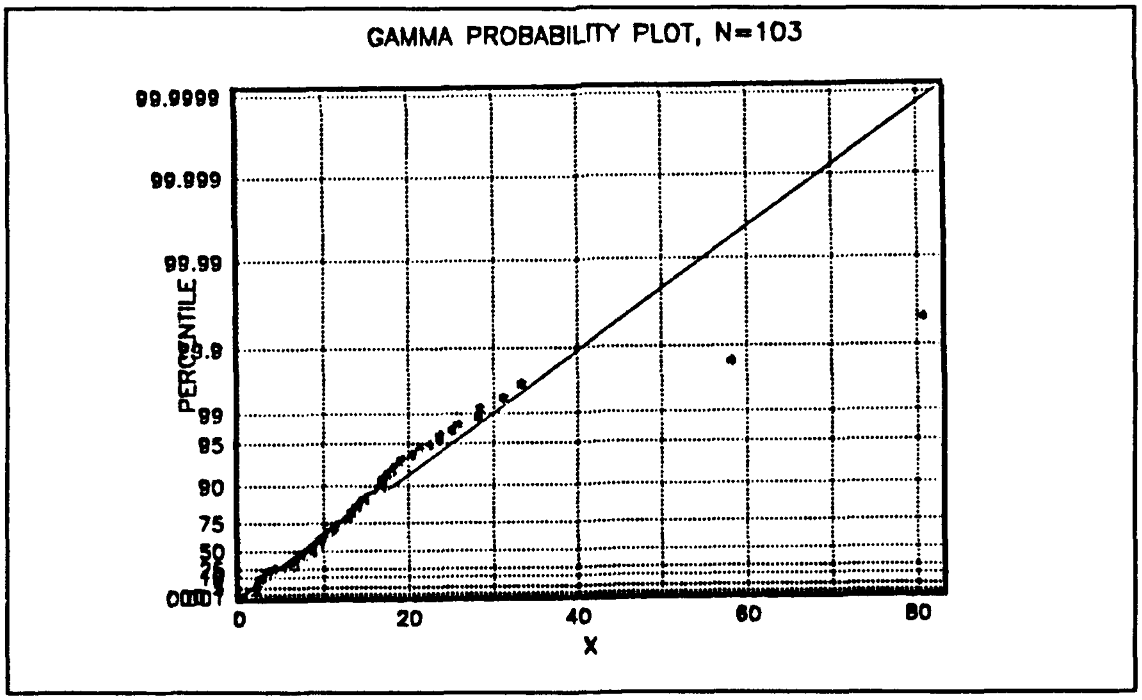

Figure 5: Gamma Curve Fit of 5g Exceedence Level

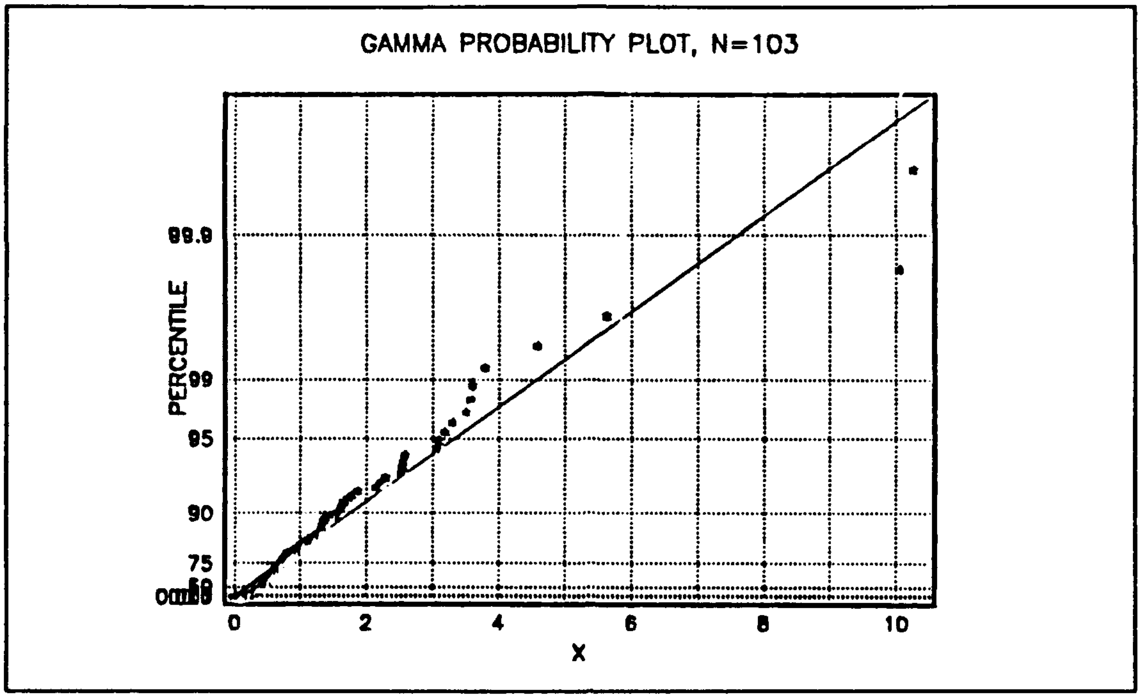

Figure 6: Gamma Curve Fit of 6g Exceedence Level

In the 7g exceedences data set, 70 of the 103 values were zero. This resulted in a large bias in the data and introduced difficulties for the curve fitting routines of AGSS. A three parameter distribution would likely have resulted in a more acceptable curve fit, but AGSS only contains MLE routines for two parameter functions. AGSS does allow the axis to be shifted along the origin, which adds a third parameter to the function. The normal procedure is to subtract the smallest value of the random variable from all the random variables in the data set, thus shifting the data to the origin. That is, if all values were between 0.8 and 6.5, 0.8 would be subtracted from each of the data points in the data set, shifting the origin of the

distribution. The x axis values on the probability plot would become x-0.8.

The values of zero in the 6g and 7g exceedence data sets precluded shifting the axis. Figure 7 displays the Weibull probability plot of the seven g data. Subtracting any value from those data sets would have generated negative values, preventing calculation of the logarithm and eliminating the use of any distribution but the normal distribution for fitting. Discarding all zeros data to the left and just fitting the remaining data points in the 7g exceedence data set still did not provide an acceptable curve fit for the data.

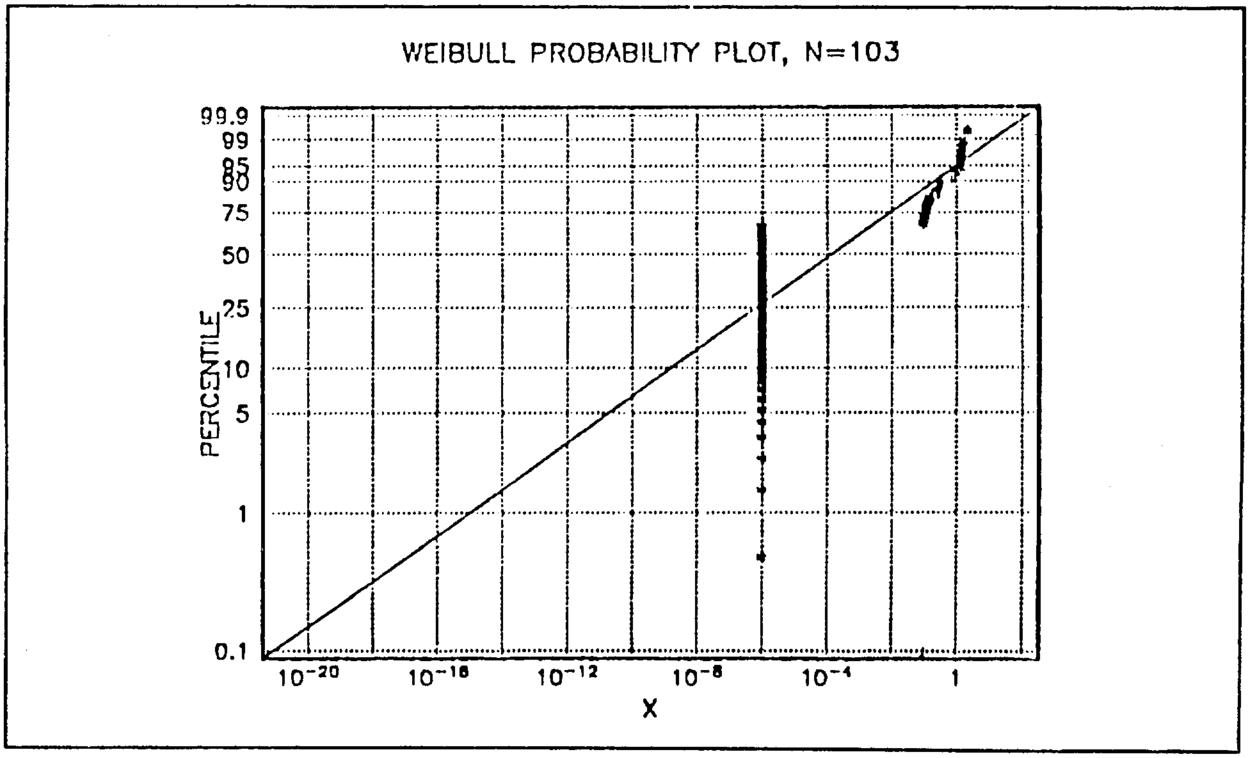

Figure 7: Seven G Exceedence Data

D. TAIL FITTING OF THE UNRESTRICTED G EXCEEDENCE DATA

1. Linear Regression

Since for the purposes of this thesis, the primary interest was in modeling the right-hand tail of the data, it was decided to censor 75% of the data to the left in all four of the g exceedence levels. Only the data from the unrestricted aircraft database was used. Censoring 75% of the data left 26, non-zero data points in each g level exceedence data set. These 26 data points were then fitted by each of the distributions, and the Weibull curve fit was the best approximation of the censored distribution of data points in the right hand tail. These data points were then plotted on a Weibull distribution probability plot and are displayed for each g exceedence level in Figures 8 through 11. The linear form of the Weibull distribution is derived in Figure 12. The left side of the linear form of the Weibull equation in Figure 12 is normally referred to as the transformed cummulative function, F* and is plotted along the right ordinate axis in Figures 8 through 11. The data was graphed using a commercial graphics program called DeltaGraph Professional and the curve fitting routines of the graphics routine provided a linear curve fit of F* as a function of the number of g exceedences per 1000 hours. The equations for each exceedence level are shown in Table 3, which immediately follows Figure 12. The probabilities along the left ordinate axis of the figures was computed by calculating F* at the various probability levels. For a probability of 0.9973, F*(0.9973)

was computed to be: ln(-ln(1-0.9973)) = 1.777, and so the 0.9973 tick mark on the left ordinate axis corresponds to a value of 1.777 on the right ordinate axis (F*).

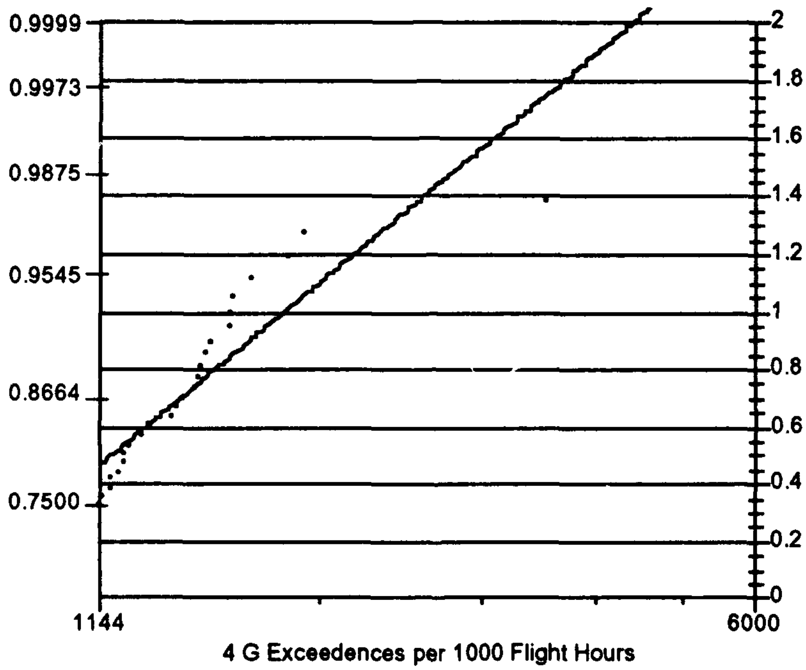

Figure 8: Weibull Curve Fit of 4 g Tail Data

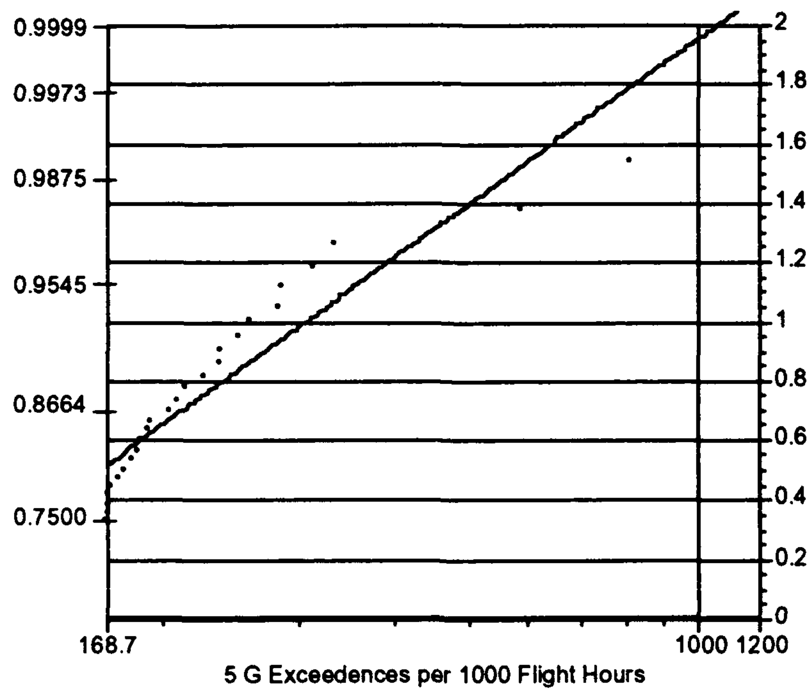

Figure 9: Weibull Curve Fit of 5 g Tail Data

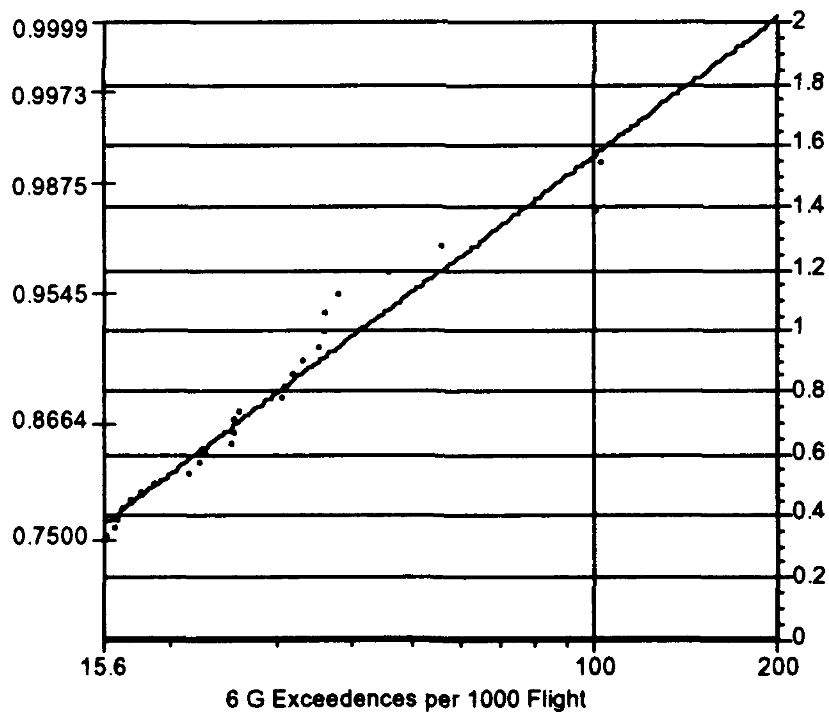

Figure 10: Weibull Curve Fit of 6 g Tail Data

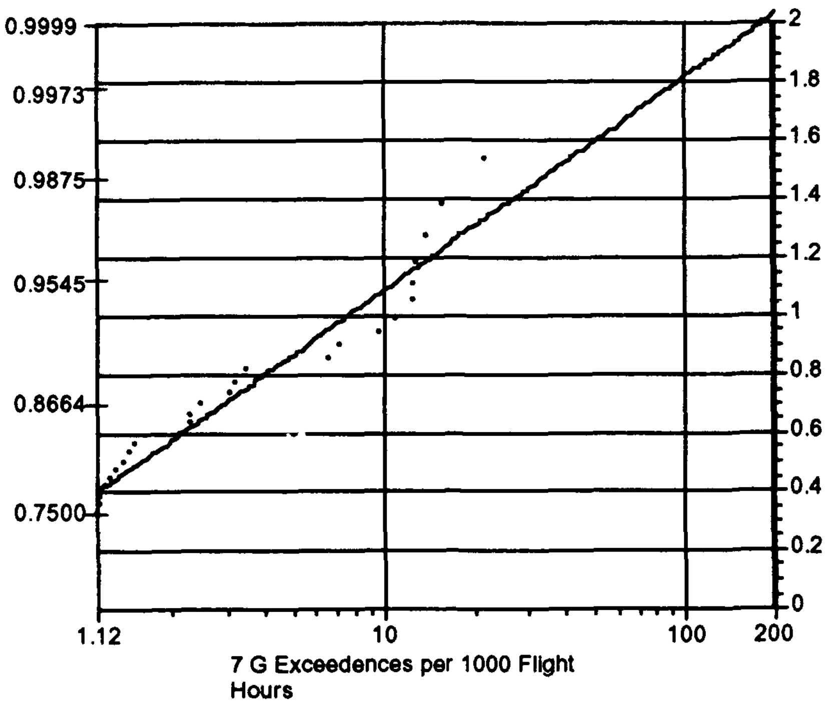

Figure 11: Weibull Curve Fit of g G Tail Data

$$F = 1 - \exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right]$$

Weibull CDF with
α=Scale Parameter
β=Shape Parameter

$$1 - F = \exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right]$$

$$\ln(1 - F) = -\left(\frac{X}{\alpha}\right)^{\beta}$$

$$-\ln(1 - F) = \left(\frac{X}{\alpha}\right)^{\beta}$$

$$\ln[-\ln(1 - F)] = \beta * \ln\left(\frac{X}{\alpha}\right)$$

Linear Form of Equation

Figure 12: Linear Form of Weibull Distribution

TABLE 3: LINEAR REGRESSION RESULTS FROM FIGURES 8 TO 11.

Exceedence Level	Equation of Line from Curve Fit Routine
4 g	$f(x) = 1.138636E+0 * \ln(x) + -7.548434E+0$
5 g	$f(x) = 8.114468E-1 * \ln(x) + -3.645683E+0$
6 g	$f(x) = 6.421064E-1 * \ln(x) + -1.385959E+0$
7 g	$f(x) = 3.153416E-1 * \ln(x) + 3.633265E-1$

2. Discussion of MLE Curve Fitting Techniques

As shown in the previous paragraphs, linear regression was used to estimate a curve fit of the last 25% of the data points for each of the four g exceedence levels. A more accurate method would be to have fit the data to the Weibull distribution, using the maximum likelihood method to calculate the best values of the parameters, α and β. This procedure is shown in Appendix B. The difficulty encountered was that the estimation of the parameters by MLE methods for a censored data set is different than the procedures for a non-censored data set. The mathematics for parameters estimation by MLE when censoring to the richt is straightforward, but the mathematics for parameter estimation when censoring to the left is difficult, and a solution to the problem could not be found. A solution could have been obtained graphically by plotting the likelihood function for various values of α and β.

The decision to utilize a linear regression (least squares) curve fit of the data is conservative in comparison to the results that would be expected using the MLE technique. The MLE method

weighs a data point by the probability density of the data. Therefore the tail of the data set is weighted less than data points nearer the centroid. The least squares method employed weights all data points equally. Thus Figure 13 shows why the least squares method utilized is a conservative approach. Since the MLE method gives points further out in the tail a smaller value for the moment than the least squares method, and since these observations deal with the right hand tail, the MLE estimation method would produce a line with less slope than the least squares method. This is illustrated in Figure 13. If using the MLE method, an aircraft at any given percentile level would experience fewer g exceedences per 1000 hours than the same aircraft based on the least squares curve fit. Thus, the Least Squares method would be expected to predict a higher number of exceedences per 1000 hours than the MLE method, and the least squares method is the more conservative of the two methods.

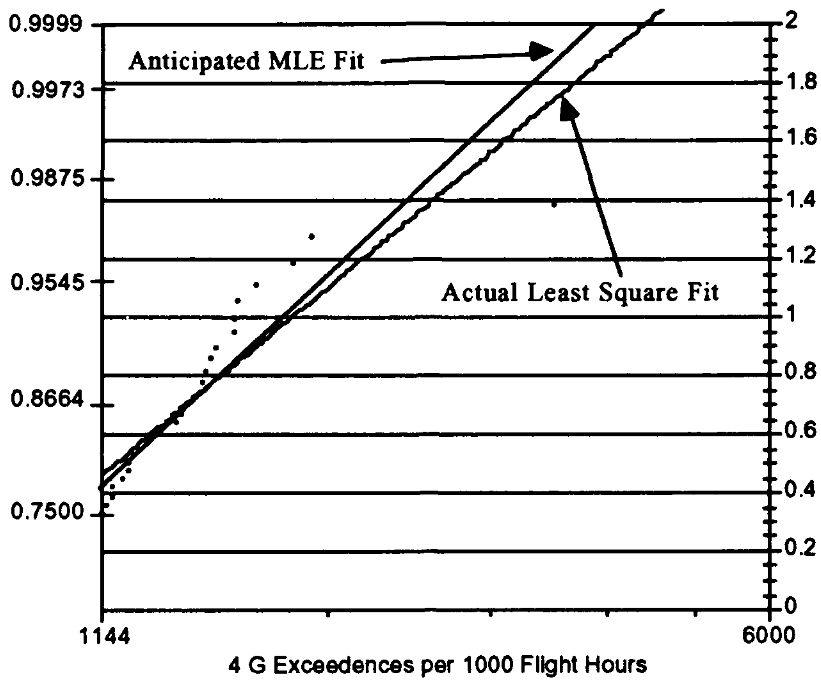

Figure 13: Comparison of Least Squares vs. MLE

IV. FLIGHT LOAD SPECTRUM DEVELOPMENT

A. EXCEEDENCES VS. G LEVEL CURVES

A graph of the number of g exceedences at each level as a function of the g level was constructed. The percentiles selected, and their associated number of g exceedences per 1000 hours expected, corresponded to multiples of standard deviations, σ, associated with the normal distribution of 1.15σ, 1.5σ, 2σ, 2.5σ, and 3σ. These σ values equate to percentiles of 0.75, 0.86638, 0.9545, 0.98758, and 0.9973 respectively. This is no way implies that the distribution of g exceedences is normal, since the curve fitting routines showed it was not normal, but there were chosen as arbitrary values to study to correspond to values typically used by NAVAIR. The number of g exceedences to be expected at each exceedence level were calculated utilizing the linear regression curve fits from Chapter III. The expected number of g exceedences at each g level for the different σ levels were summarized in Table 4. The results were also displayed graphically in Figure 14, although the slight variation in the data and the resolution of the graph made distinction of the different σ levels in the graph difficult.

TABLE 4: EXPECTED NUMBER OF G EXCEEDENCES

G Level	Sigma Level	Probability Level	Expected Number of G Exceedences
4G	1.15	0.75	1644.44
4G	1.5	0.86638	1814.33
4G	2	0.9545	1928.41
4G	2.5	0.98758	1968.53
4G	3	0.9973	1980.07
5G	1.15	0.75	267.24
5G	1.5	0.86638	301.05
5G	2	0.9545	323.75
5G	2.5	0.98758	331.73
5G	3	0.9973	334.03
6G	1.15	0.75	35.78
6G	1.5	0.86638	41.70
6G	2	0.9545	45.67
6G	2.5	0.98758	47.07
6G	3	0.9973	47.47
7C	1.15	0.75	6.71
7G	1.5	0.86638	8.46
7G	2	0.9545	9.63
7G	2.5	0.98758	10.05
7G	3	0.9973	10.16

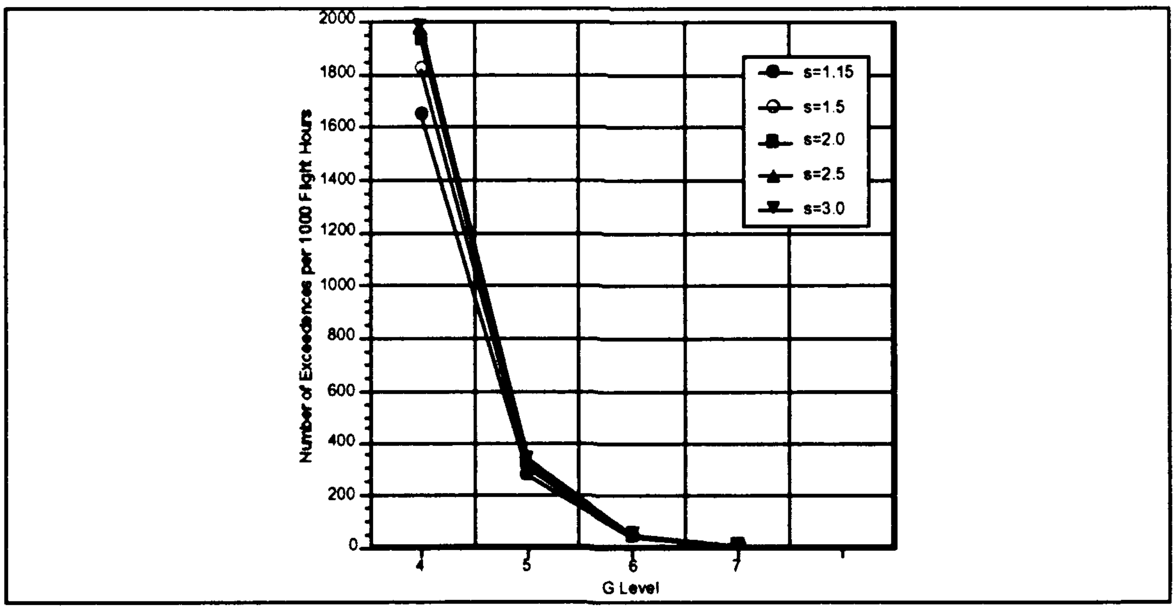

Figure 14: Number of G Exceedences per σ Level per G Level

B. ANTICIPATING INTERMEDIATE G LEVELS

It was desired to estimate the number of intermediate g levels that an aircraft might experience. When the aircraft performs a maneuver and the counting accelerometer records an exceedence at one of the levels, say 4g's the actual g experienced by the aircraft may be anywhere between 4.0 and 5.0 g's. The actual g felt by the aircraft was estimated by a log-linear graph of the number of g exceedences per g level displayed in Figure 15. Since the plot of number of g exceedences vs. the various g exceedence levels can be linearized on a semi-log plot, it was assumed that the distribution of peak g levels between each of the g levels would fall on that line. That is to say that the distribution of maximum g levels between the 4.0 and 5.0 g levels would fall on the log-linear line. Since the line through the different g levels was approximately straight and parallel for all of the different sigma levels, the line was linearly fit in the semi-log space for one set of data and utilized for all of the g levels at the various sigma levels.

The curve fit of the log-linear plot of Figure 15 was displayed in Equation 1. Values for the x were varied from 4.0 to 5.0 in 0.1

$$\ln(y) = -1.7633 * x + 14.63 \qquad (1)$$

increments and values for y calculated. The calculated values for y were corrected by subtraction of the lowest value so that the y values could be normalized. After normalization, they were summed

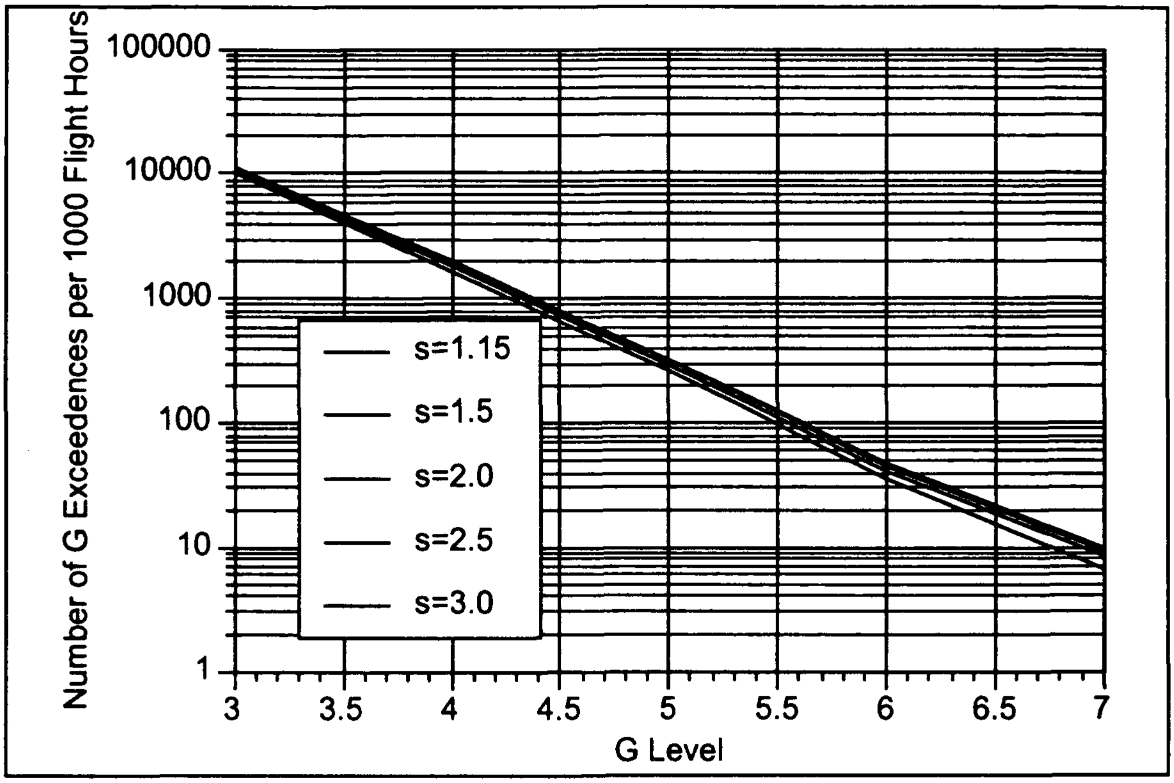

Figure 15: Number of G Exceedences per σ Level per G Level

in a cumulative manner. These calculations were summarized in Table 5. The summation column of Table 5 and a random number generator in the computer program were utilized to select the intermediate g level obtained. For example, if the random number generator returned a value of 0.18, from the summation columns in Table 6, it is seen that 0.18 is less than the summation value for 4.0 g's, so the expected g level was 4.0 g's. If the random number generator returned a value between 0.195 (4.0 g level) and 0.359 (4.1 g level) then an intermediate g level of 4.1 would be expected, etc. The same routine was utilized for predicting the immediate g

levels for the 4.0g to 4.9g level, 5.0g to 5.9g levels, 6.0g to 6.9g levels, and the 7.0g to 7.9g levels.

TABLE 5: SUMMARY OF CALCULATIONS FOR GENERATION OF INTERMEDIATE G LEVELS

X	Y	Y CORRECTED	PERCENTILE	SUMMATION
4.0	1952.4	1617.6	0.195	0.195
4.1	1636.8	1302.0	0.164	0.359
4.2	1372.2	1037.4	0.137	0.496
4.3	1150.3	815.5	0.115	0.611
4.4	964.4	629.6	0.096	0.707
4.5	808.5	473.7	0.081	0.788
4.6	677.8	343.0	0.068	0.856
4.7	568.2	233.4	0.057	0.912
4.8	476.4	141.6	0.048	0.960
4.9	399.3	64.5	0.040	1.000
5.0	334.8	0.0	0.000	

Since NAVAIR includes three g exceedences in their standard load spectrum for the A-6, the data displayed graphically in Figure 15 was utilized to estimate the number of 3g exceedences that could be expected from the results of this thesis. Plots were made at each of the sigma levels of the number of exceedences per 1000 hours as a function of the four different g levels. Linear regression analysis was performed via the use of DeltaGraph Professional and the number of 3g exceedences were summarized in Table 6.

TABLE 6: ESTIMATED NUMBER OF 3G EXCEEDENCES

Sigma Level	Expected 3G Exceedences
1.15	10378
1.5	10823
2.0	11181
2.5	11309
3.0	11353

C. GENERATION OF A LOAD SEQUENCE

It was desired to create a 1000 hour, cycle by cycle, flight load sequence history from the data for use in the fatigue calculations. This was generated by utilizing the results of the statistical analysis and a load sequence from a typical aircraft in the database. The statistical analysis provided the number of exceedences to be expected at each level, and the typical aircraft's load sequence was modified to provide a typical order of sequence for the application of the loads.

Since no data was available that would describe the cycle by cycle sequence for the A-6, one was generated from a typical unrestricted aircraft in the data base. Bureau number 162190 was chosen from the data base to generate a typical 1000 hour cycle by cycle sequence. The load spectrum of aircraft 162190 was actually less severe than the average aircraft in the database. This can be verified by calculating the average number of g exceedences per 1000 hours of flight time for aircraft 162190 and comparing it to the average statistics listed in Chapter I. This was not important in this analysis since the topic of interest is only the sequence of the applied loads and not the load spectrum of aircraft 162190. Later corrections to the generated load sequence will provide the proper number of exceedences at each level.

Table 7 displays the recorded load history of bureau number 162190. The data in Table 7 indicated the number of g exceedences encountered at each g level on a monthly basis. Since the aircraft

selected represents a load history of only 654.5 hours, all values for the numbers of g exceedences were corrected by a factor of 1000/654.4. The corrected 1000 hour load history is displayed in Table 8.

TABLE 7: LOAD HISTORY OF BUREAU NUMBER 162190

Date of Data Report	Flight Hours	4G Exceedences	5 G Exceedences	6G Exceedences	7G Exceedences
12/31/89	47.6	20	5	0	0
1/31/90	31.9	23	2	0	0
2/28/90	22.5	9	0	0	0
3/31/90	10.7	9	1	0	0
4/30/90	26.5	29	4	0	0
5/31/90	32.9	11	1	0	1
6/30/90	51.0	4	1	0	0
7/31/90	22.2	5	0	0	0
8/31/90	52.6	16	0	0	0
9/30/90	29.1	4	0	0	0
10/31/90	67.8	11	1	0	0
11/30/90	28.0	14	0	0	0
12/31/90	19.3	3	0	0	0
7/31/91	2.7	13	0	0	0
8/31/91	50.8	7	0	0	0
9/30/91	48.8	5	1	0	0
10/31/91	55.8	32	1	0	0
11/30/91	54.3	28	5	1	0
Total Hours	654.5				

The next step in creating a cycle by cycle sequence was to correct the number of exceedences at each g level to the desired level. Load sequences were created for each of the sigma levels discussed in Chapter III. For illustration, a cycle by cycle sequence for a 0.9973 percentile (three sigma) aircraft will be developed. As presented in Chapter III, the expected number of exceedences for a 99.73 % aircraft at the 4g, 5g, 6g, and 7g, levels were 1980.07, 334.03, 47.47, and 10.16 respectively. To generate the 3σ load history of exceedences, the number of g exceedences at each of the g

levels was multiplied by the number of expected g exceedences for a 3σ spectrum and then divided by the total number of exceedences at each level from the corrected 1000 hour load history. The process

TABLE 8: CORRECTED 1000 HOUR LOAD HISTORY

Date of Data	Corrected FLt. Hours	4G PER 1000.0	5G PER 1000.0	6G PER 1000.0	7G PER 1000.0
12/31/89	72.7	31.	8.	0.	0.
1/31/90	48.7	35.	3.	0.	0.
2/28/90	34.4	14.	0.	0.	0.
3/31/90	16.3	14.	2.	0.	0.
4/30/90	40.5	44.	6.	0.	0.
5/31/90	50.3	17.	2.	0.	2.
6/30/90	77.9	6.	2.	0.	0.
7/31/90	33.9	8.	0.	0.	0.
8/31/90	80.4	24.	0.	0.	0.
9/30/90	44.5	6.	0.	0.	0.
10/31/90	103.6	17.	2.	0.	0.
11/30/90	42.8	21.	0.	0.	0.
12/31/90	29.5	5.	0.	0.	0.
7/31/91	4.1	20.	0.	0.	0.
8/31/91	77.6	11.	0.	0.	0.
9/30/91	74.6	8.	2.	0.	0.
10/31/91	85.3	49.	2.	0.	0.
11/30/91	83.0	43.	8.	2.	0.
Totals	1000	373	37	2	2

for each of the g levels is illustrated in the equations in Figure 16. The calculated 3σ load history was displayed in Table 9.

$$4g.\,correction = (\#\,of.\,exceedences) * \frac{1980.07}{373}$$

$$5g.\,correction = (\#of.\,exceedences) * \left(\frac{334.03}{37}\right)$$

$$6g.\,correction = (\#\,of.\,exceedences) * \left(\frac{47.47}{2}\right)$$

$$7g.\,correction = (\#\,of.\,exceedences) * \left(\frac{10.16}{2}\right)$$

Figure 16: Correction for Transfer to 3σ Spectrum

TABLE 9: THREE σ LOAD SPECTRUM

Date of Data	Corrected FLt. Hours	4G PER 1000.0	5G 1000.0	6G 1000.0	7G 1000.0
12/31/89	72.7	165	72	0	0
1/31/90	48.7	186	27	0	0
2/28/90	34.4	74	0	0	0
3/31/90	16.3	74	18	0	0
4/30/90	40.5	234	54	0	0
5/31/90	50.3	90	18	0	10
6/30/90	77.9	32	18	0	0
7/31/90	33.9	42	0	0	0
8/31/90	80.4	127	0	0	0
9/30/90	44.5	32	0	0	0
10/31/90	103.6	90	18	0	0
11/30/90	42.8	111	0	0	0
12/31/90	29.5	27	0	0	0
7/31/91	4.1	106	0	0	0
8/31/91	77.6	58	0	0	0
9/30/91	74.6	42	18	0	0
10/31/91	85.3	260	18	0	0
11/30/91	83.0	228	72	47	0
Totals	1000	1978.0	333.0	47.5	10.2

While Table 9 provides the number of g exceedences that a typical aircraft might experience in a month by month sequence, it provides no information as to the order of g applications within a given month. Therefore a random number generator was utilized to create the load sequence within each month. A FORTRAN code was written that worked month by month through the predicted 1000 hour load history displayed in Table 9. The computer algorithm would select a random number and then pick a corresponding 4g, 5g, 6g, or 7g load. To illustrate the first line of monthly data was used from Table 9. The first line of data only has exceedences at the 4g and 5g level. There were a total of 165+72=237 exceedences to be experienced during the first month of the sequence with 165 (69.62%) of them at the 4g level and 72 (30.38%) of them at the 5g level. A random number generator provided a random number

between zero and one, and if the number were below 0.6962 then a 4g load was inserted into the load sequence and if the random number generator choose a number larger that 0.6962 then a 5g load was inserted into the sequence. If all the loads were depleted at a particular g level, and a random number was generated picking a depleted g level, then another random number was called until all the exceedences had been picked from the four different g levels. When all the bins had been emptied for a particular month, the program proceeded in the same fashion to the next month.

As each random number was generated and a g level exceedence chosen, the program immediately called another random number and used the log-linear relationship developed in Section B of this chapter to determine the tenths value of the load to be inserted into the sequence. For example, if a 5g load were picked to be inserted into the sequence, then the second random number would determine if it were a value of 5.0, 5.1, 5.2, etc. up to 5.9. The generated values were then sequenced into an output file in a format usable by the FAMS fatigue analysis program discussed in the following chapter. Figure 17. displays the first few lines of a sample 3σ cycle by cycle sequence created by this method. The first three lines of the data file are administrative, or provide information to the FAMS fatigue calculation program. Lines four and following are the generated cycle by cycle sequences. A graphical representation of the first twenty load sequences from the Figure 17 data was presented in Figure 18.

The value of 1.0g was chosen as the minimum value of each load cycle for the cycles in the load spectrum. During normal maneuvering, an aircraft will experience less than one g of acceleration some percentage of the time as the pilot reduces the acceleration on the airplane back to the one g level following a maneuver. This would result in a more severe fatigue spectrum, but is not a necessary consideration for the comparison study performed here. The emphasis and interest in this thesis is the effect of changes in the peak loads of the fatigue spectrum and the valley value of one g was chosen as a reference.

```
     1     'CYCLE'
FILE NUMBER IS  3   RAND=2222  3.0 SIGMA AIRCRAFT
          2369         1000.00
  5.20     1.00     4.10     1.00     4.30     1.00     5.50     1.00
  4.10     1.00     5.10     1.00     4.50     1.00     4.10     1.00
  4.40     1.00     4.50     1.00     5.30     1.00     5.10     1.00
  4.80     1.00     4.80     1.00     4.80     1.00     4.20     1.00
  5.20     1.00     4.20     1.00     5.20     1.00     4.70     1.00
  5.80     1.00     4.00     1.00     4.00     1.00     4.10     1.00
  4.70     1.00     4.10     1.00     4.00     1.00     4.00     1.00
  4.40     1.00     4.00     1.00     4.40     1.00     4.30     1.00
  4.10     1.00     5.20     1.00     4.80     1.00     5.10     1.00
  4.00     1.00     4.10     1.00     5.00     1.00     5.30     1.00
```

Figure 17: Sample 3σ Cycle by Cycle Sequence File

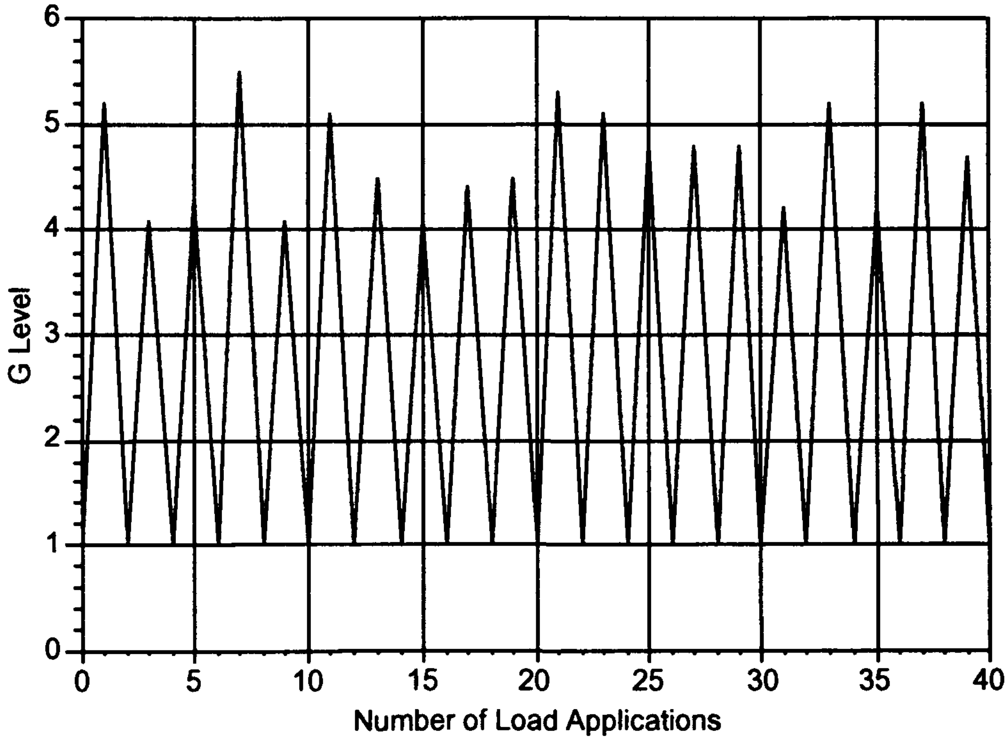

Figure 18: Cycle by Cycle Load Application

V. FATIGUE ANALYSIS

A. FAMS FATIGUE ANALYSIS PROGRAM

FAMS is a computer algorithm written by N. R. Krishnan of Aerostructures, INC. The program calculates damage and remaining fatigue life for various metals. Program output includes the calculated damage and fatigue life remaining in hours. Data can be input in either a cycle by cycle format or a block format. Material properties are input in formats that allow the program to produce stress-strain and strain-life curves.

The FAMS algorithm used a strain-life approach that uses the Neuber relationship of Equation 2 to calculate the local stresses and strains at the notch during the damage calculations. The Neuber notch factor, K_N, is the fatigue stress concentration and is related to the K_t, the geometric stress concentration factor, by Equation 3. The

$$\boxed{K_N^2 = K_\sigma * K_\varepsilon} \qquad (2)$$

$$\boxed{K_N = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{a}{m}}}} \qquad (3)$$

radius of the notch root is m, and a is a characteristic dimension of the material. The mean stress is accounted for by use of an equivalent strain relationship. The damage is cumulated cycle by cycle via the linear relationship of Miner's Rule.

S_{ref} is the factor that the input load sequence must be multiplied by in order to obtain the actual stress at the fatigue location of interest. This reference stress is not to be confused with the reference stress that NAVAIR uses to identify a reference stress for a particular aircraft. NAVAIR's definition of the reference stress is the local stress at the notch tip for a location of interest on the A-6 during a 6.5g symmetric maneuver. (Limit load)

For the purposes of this study, it was decided to vary the NAVAIR reference stress from 35000 PSI to 100,000 PSI. To achieve the desired range of the NAVAIR reference stress, the appropriate values of S_{ref} and K_N had to be calculated to produce these results. Inputs of S_{ref} and K_N into the FAMS algorithm are multiplied together to produce the stress applied to the material at the one g level. Since NAVAIR measures their reference stress at the 6.5 g level, the following equation was used to calculate the desired values of the FAMS reference stress. The following equation can be used to calculate NAVAIR's definition of the far field reference stress.

$$\text{Navair reference stress} = 6.5 * K_N * (\text{FAMS Sref})$$

The factor of 6.5 in the equation originates from the fact that the load factor for the A-6 is 6.5 g's. S_{ref} and K_N are input into the FAMS program.

The FAMS program also allows an input for any residual stress assumed to be at the fatigue location of interest. This feature is useful when tracking the fatigue life expended on a material that is tracked over a long period of time.

B. VARIATION OF MATERIAL PROPERTIES

It was desired to analytically study the effect on the fatigue life of an aluminum structure that is stronger or weaker than the average specimen. The material utilized for the analysis during all of the calculations was 7075-T651 aluminum. The standard material properties required for fatigue calculations were available in the FAMS material properties database The material properties in the FAMS program were modified to simulate specimens with material properties at the 10, 30, 50, 70, 90, 95.5, and 99.7 percentile levels. The 50% specimen is considered the specimen with the standard properties. A 10% specimen would have the properties at least as strong as the tenth specimen in a ranked (weakest to strongest) population of 100 specimens that had been tested to failure.

The methods utilized to modify the material properties were taken from a paper by Sinclair and Dolan. [Ref. 5] Sinclair and Dolan performed a large number of fatigue tests for 7075-T651 at different stress levels. They were able to show that the material properties varied from a 50% specimen by Equation 4, where S is the stress and σ is the standard deviation.

$$\boxed{\sigma = (8.72E13) * S^{-3.19}} \qquad (4)$$

The strain-life equation for the strain used by the FAMS program for N greater than 500 cycles to failure, is shown in Equation 5. N is the number of cycles and ε is the strain amplitude. Using the stress-strain relationships from the FAMS program, shown in Equation 6, stresses were calculated for a range of cycles.

$$\boxed{\varepsilon = \frac{0.02035}{N^{0.1573}} + \frac{565.860}{N^{3.1484}}} \qquad (5)$$

$$\boxed{\varepsilon = \frac{S}{10.3E6} + 1.0956E-16*(S-68000)^{3.27}} \qquad (6)$$

Then the standard deviation on N was calculated for each of the stress levels using Equation 4. Percentiles of 10%, 30%, 70%, 90%, 95.5%, and 99.7% were selected, which corresponded to shifts in the number of cycles from the mean specimen, of -1.28126σ, -0.5224σ, 0.5244σ, 1,28126σ, 2σ, and 3σ respectively. As an example, the results for a 70% specimen are displayed in Table 10.

TABLE 10: RESULTS OF CALCULATIONS FOR A 70% SPECIMEN

Stress	Log N	+0.2544σ	log N_{70}
78979	2.6990	0.0109	2.7099
72078	3	0.0146	3.0146
68464	3.1239	0.0172	3.1411
49224	4	0.0492	4.0492
35268	5	0.1563	5.1563
23855	6	0.4963	6.4963

Then the stress was used to calculate the elastic and plastic strain components at each of the percentile levels. Again the results for a 70% specimen are shown in Table 11.

TABLE 11: RESULTS OF STRAIN CALCULATIONS FOR 70% SPECIMEN

$\log \varepsilon_e$	$\log \varepsilon_p$	$\log N_{70}$
-2.1153	-2.7476	2.7099
-2.1379	-3.3917	2.8880
-2.1550	-4.1549	3.0146
-2.1774	-	3.1411
-2.3207	-	4.0492
-2.4779	-	5.1563
-2.6353	-	6.4963

The elastic and plastic strain values were then plotted as a function of the number of cycles to fatigue failure on log-log coordinates. A linear regression provided the coefficients required for the FAMS fatigue analysis program to model the various percentile materials. The required FAMS coefficients are those shown in Equations 7 and 8, and the coefficients used for the various materials during the fatigue calculations are shown in Table 12.

$$\varepsilon_e = \frac{C_1}{N^{B_1}} \qquad (7)$$

$$\varepsilon_p = \frac{C_2}{N^{G_2}} \qquad (8)$$

TABLE 12: SUMMARY OF FAMS COEFFICIENTS REQUIRED FOR ANALYSIS

Percentile	C_1	B_1	C_2	G_1
10	.0320	.2235	9.8090 E09	4.7484
30	.0243	.1819	6.9513 E09	4.6652
50	.0204	.1573	5.4946 E09	4.6085
70	.0183	.1404	4.3670 E09	4.5533
90	.0156	.1183	3.1811 E09	4.4766
95.5	.0142	.1041	2.3627 E09	4.4052
99.7	.0126	.0880	1.5969 E09	4.3105

The elastic and plastic strains for all the percentile and cycle levels were added to obtain the total strain of the various percentile materials. A graphical representation of the results were presented in Figure 19. It can be observed in Figure 19, that the deviation of the material properties is greater at lower stress levels, which agrees with the results of Sinclair and Dolan.

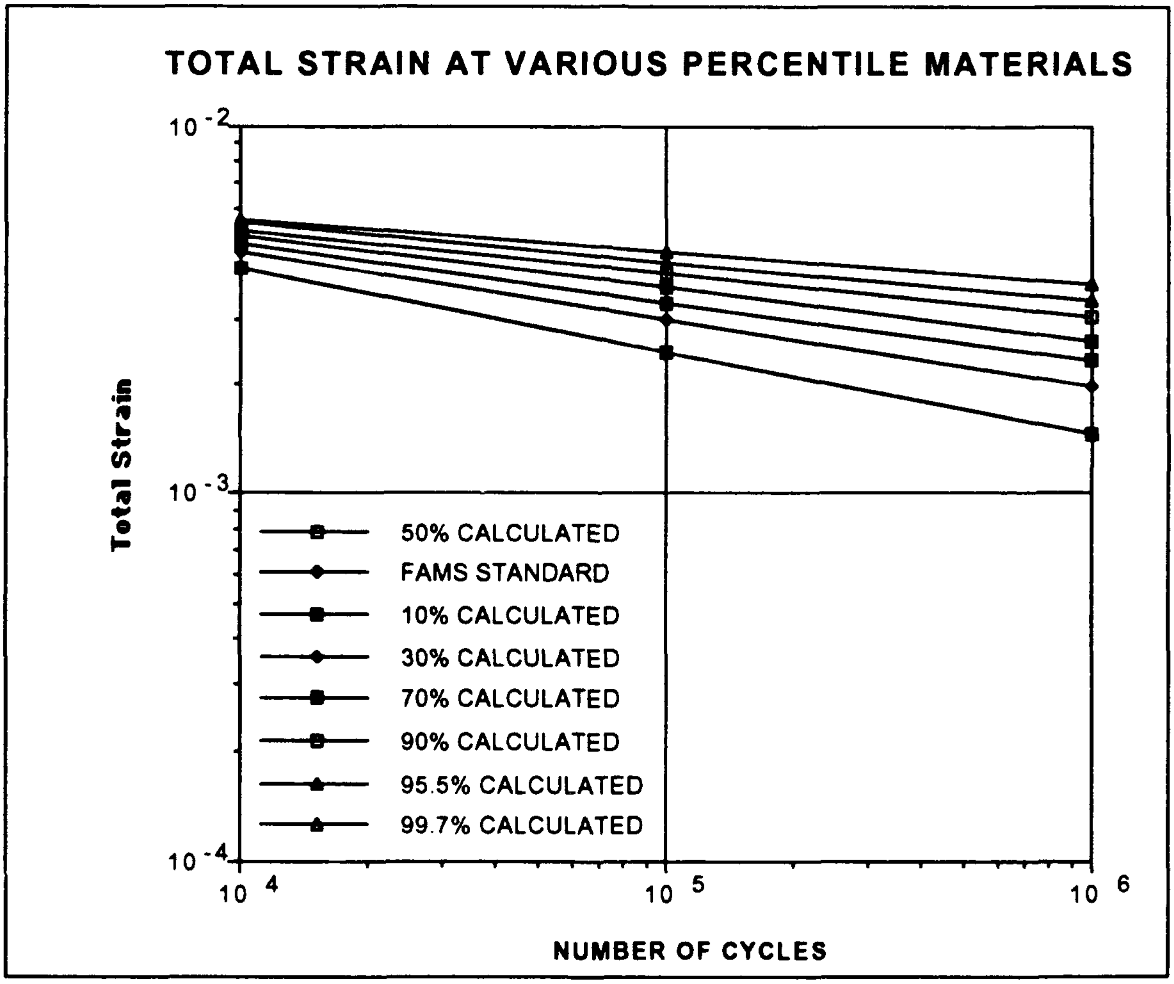

Figure 19: Total Strain for Various Percentile Materials

C. SENSITIVITY STUDIES

Sensitivity studies were carried out to study the effects of varying the random number used to produce the cycle by cycle load spectrum and the effects of varying the NAVAIR reference stress. The fatigue life was also calculated at each of the different aircraft percentile spectrums. All results of fatigue life calculations output from the FAMS program were divided by a factor of four to allow for the analytical factor of safety of four used by NAVAIR. The metal used for all fatigue calculations was 7075-T651.

Additionally, the effect of varying the 7075-T651 fatigue properties on the fatigue life of the A-6 was studied. This was done to investigate the effects of manufacturing variations on the fatigue life at aluminum specimens

1. Varying the Cycle by Cycle Sequence

It was desired to study the effect on the fatigue life calculations of varying the randomized portion of the cycle by cycle sequence on the fatigue life calculations. The cycle by cycle sequence at the 2.5σ and 3.0σ levels were varied by changing the random number seed in the FORTRAN algorithm that produced the sequences. Nine runs were made at the 2.5σ and 3.0σ levels with different cycle by cycle sequences while holding the S_{ref} and K_N values constant. The results were listed in Table 13. Randomization of the cycle by cycle sequences resulted in varying the remaining life by no more than 8.9% from the mean for the case of the 3σ sequence and no more than 9.4% for the 2.5σ sequence. As expected, the average life

remaining at the 2.5σ level is greater than the average life remaining at the 3.0σ level. The difference in life remaining between the 2.5σ and 3.0σ sequences was so small that individual runs with the FAMS program resulted in 2.5σ sequences being more severe that the 3.0σ calculations. This was caused by the random number generation techniques used to develop the cycle by cycle sequences. The difference between the average life remaining calculations for the 2.5σ and 3.0σ levels was only 6.5%, while the random number generated sequences varied as much as 12.7% (run 7, last column)

TABLE 13: LIFE SENSITIVITY TO REORDERED LOADS IN THE SEQUENCE

Run #	Life Remaining 3.0 Sigma	% from Mean 3.0 Sigma	Life Remaining 2.5 Sigma	% from Mean 2.5 Sigma	ABS(Column 3 Minus Column 5)
1	4851658	2.7788%	5199705.	3.5227%	0.7439%
2	5188580	-3.9727%	4858853.	-3.2635%	0.7092%
3	4545068	8.9225%	4872855.	-2.9847%	11.9072%
4	5088758	-1.9724%	5146298.	2.4594%	4.4318%
5	4935620	1.0963%	4550450.	-9.4036%	10.4999%
6	4814908	3.5152%	5307778.	5.6743%	2.1591%
7	5298813	-6.1816%	5349093.	6.4969%	12.6785%
8	4812848	3.5565%	5033900.	0.2216%	3.3349%
9	5376713	-7.7426%	4885988.	-2.7232%	5.0194%
AVG's	**4990329**	**0.0000%**	**5022769**	**0.0000%**	

2. Varying the Reference Stress

To study the effect of changing the NAVAIR reference stress on the fatigue life, calculations were performed that varied the NAVAIR reference stress between 35 KSI and 100 KSI. The values of K_N and S_{ref} used to achieve the desired NAVAIR reference stress were displayed in Table 14. A 500 hour block of load data provided

by NAVAIR was used as for the load sequence during all runs. This was the same block load utilized by NAVAIR to define the expected g loads to be encountered on a 3σ A-6. The calculated life remaining, in hours, for the various NAVAIR reference stresses were listed in Table 14 and displayed graphically in Figure 20. The ordinate axis of the plot in Figure 20 used a logarithmic scale to provide a linear representation of the data. The plot is linear on a Log-log plot up to the calculated reference stress of 100 ksi.

TABLE 14: EFFECT ON LIFE BY VARYING NAVAIR REFERENCE STRESS

Nueber Notch Factor	FAMS Sref	NAVAIR Reference Stress	Remaining Life
2.5	2153.846	35000	46626300
2.5	2307.692	37500	8756670
2.5	2461.538	40000	5247633
2.5	2615.385	42500	3218398
2.5	2769.231	45000	1072238
2.5	2923.08	47500	690876
2.5	3076.923	50000	452311
2.5	3230.769	52500	298770
2.5	3384.615	55000	201954
2.5	3538.462	57500	115807
2.5	3692.308	60000	79978
2.5	3846.154	62500	55728
2.5	4000.000	65000	39144
2.5	4153.846	67500	27693
2.5	4307.692	70000	19468
2.5	4461.538	72500	14121
2.5	4615.385	75000	10171
2.5	4769.231	77500	7373
2.5	4923.077	80000	5393
2.5	5076.923	82500	4114
2.5	5230.769	85000	2973
2.5	5384.615	87500	2256
2.5	5538.462	90000	1737
2.5	5692.308	92500	1341
2.5	5846.154	95000	1047
2.5	6000.000	97500	831
2.5	6153.846	10000	670

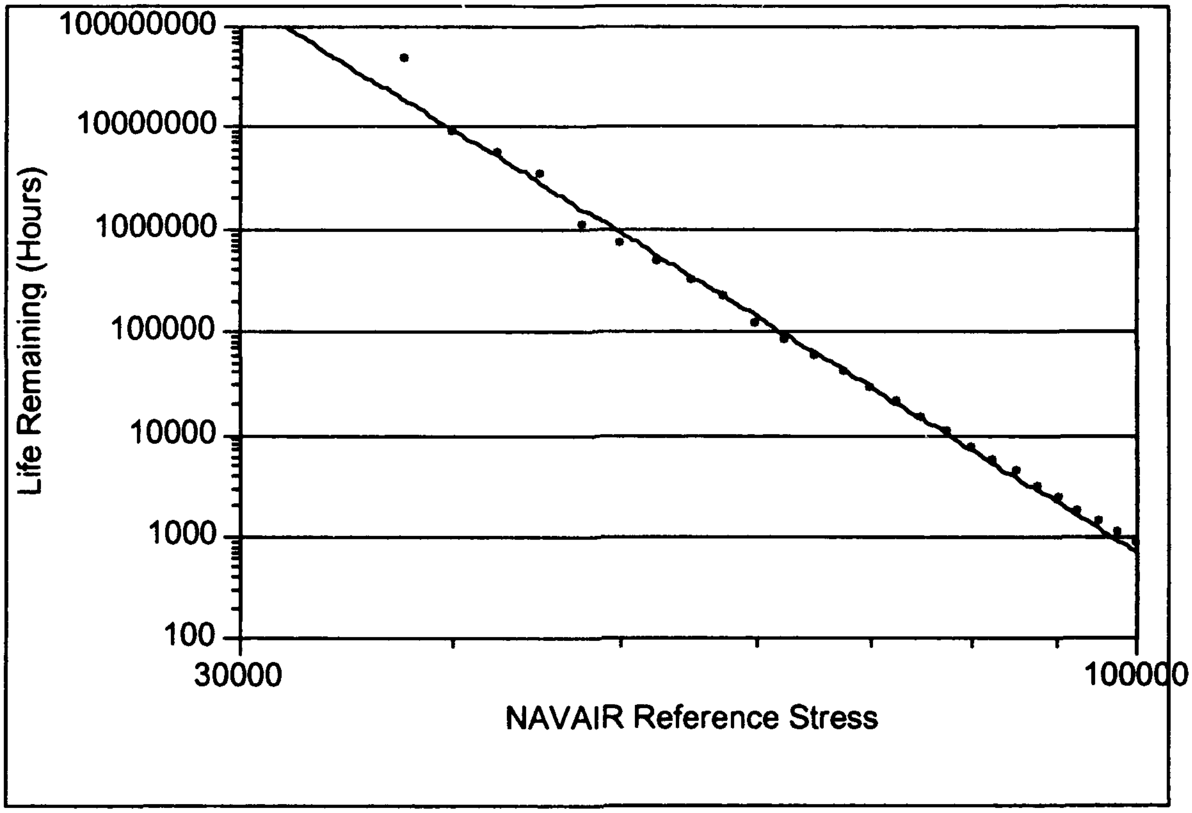

Figure 20: Effect on Life by Varying NAVAIR Reference Stress

3. Fatigue Life Versus Spectrum Percentile

Calculations were performed to measure the damage for the different percentile spectrums for the aircraft. The life remaining calculations were an average of nine different runs using different cycle by cycle sequences for each of the calculations. The several cycle by cycle sequences were generated by using different random number seeds during sequence generation. The results for a reference stress of 40 ksi, 55 ksi, and 75 ksi were displayed in Tables 15-17. The last column of the tables indicates the percentage

change in life remaining from the 3.0σ level. The change in the life of the specimen between the 3.0σ specimen and the 2.5σ and 2.0σ specimen was not significant. The difference in life between the 3.0σ and the 1.5σ specimen was very large. A graphical representation of the data was presented in Figures 21, 22, and 23. The data was not linear on a log-linear plot or a log-log plot. Due to randomization effects, the 2.5σ spectrum had a shorter life than the 3.0σ spectrum during some calculations. This was most observable on the plot of the 75 KSI data.

TABLE 15: CALCULATED LIFE FOR VARIOUS σ LEVELS (40 KSI)

SIGMA LEVEL	Neuber NOTCH FACTOR	FAMS S_{ref}	A-6 REFERENCE STRESS	Average Life Remaining	Percent Change From 3σ Sequence
1.15	2.5	2461.538	40000.	6738510	35.03%
1.50	2.5	2461.538	40000.	5870154	17.63%
2.00	2.5	2461.538	40000.	5120851	2.62%
2.50	2.5	2461.538	40000.	5022769	0.65%
3.00	2.5	2461.538	40000.	4990329	N/A

TABLE 16: CALCULATED LIFE FOR VARIOUS σ LEVELS (55 KSI)

SIGMA LEVEL	Neuber NOTCH FACTOR	FAMS S_{ref}	A-6 REFERENCE STRESS	Average Life Remaining	Percent Change From 3σ Sequence
1.15	2.5	3384.615	55000.	257837	23.4%
1.50	2.5	3384.615	55000.	238611	21.7%
2.00	2.5	3384.615	55000.	214046	2.42%
2.50	2.5	3384.615	55000.	207906	-0.5%
3.00	2.5	3384.615	55000.	208972	N/A

TABLE 17: CALCULATED LIFE FOR VARIOUS σ LEVELS (75 KSI)

Sigma Level	Neuber Notch Factor	FAMS S_{ref}	NAVAIR Reference Stress	Life Remaining	Percent Change From 3s Sequence
1.15	2.5	4615.385	75000	24619	0.23375471
1.5	2.5	4615.385	75000	21158	0.06030795
2	2.5	4615.385	75000	19228	-0.0364083
2.5	2.5	4615.385	75000	19269	-0.0343611
3	2.5	4615.385	75000	19954	0

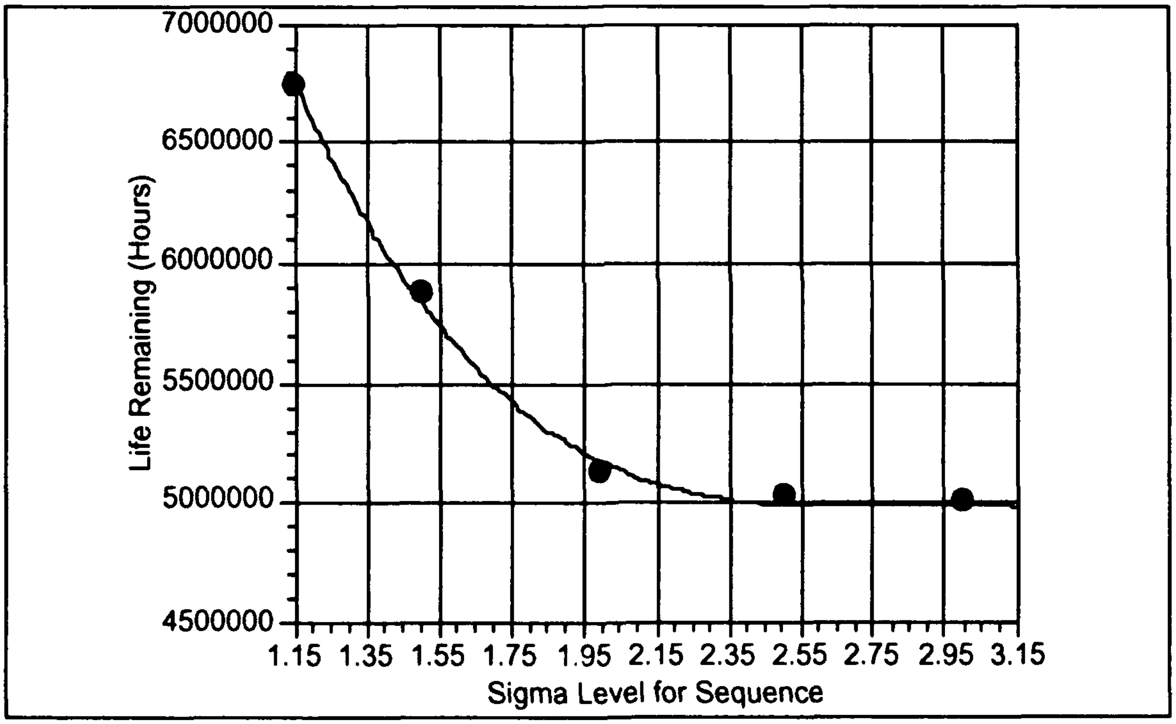

Figure 21: Life Remaining for Various σ Levels (40 KSI)

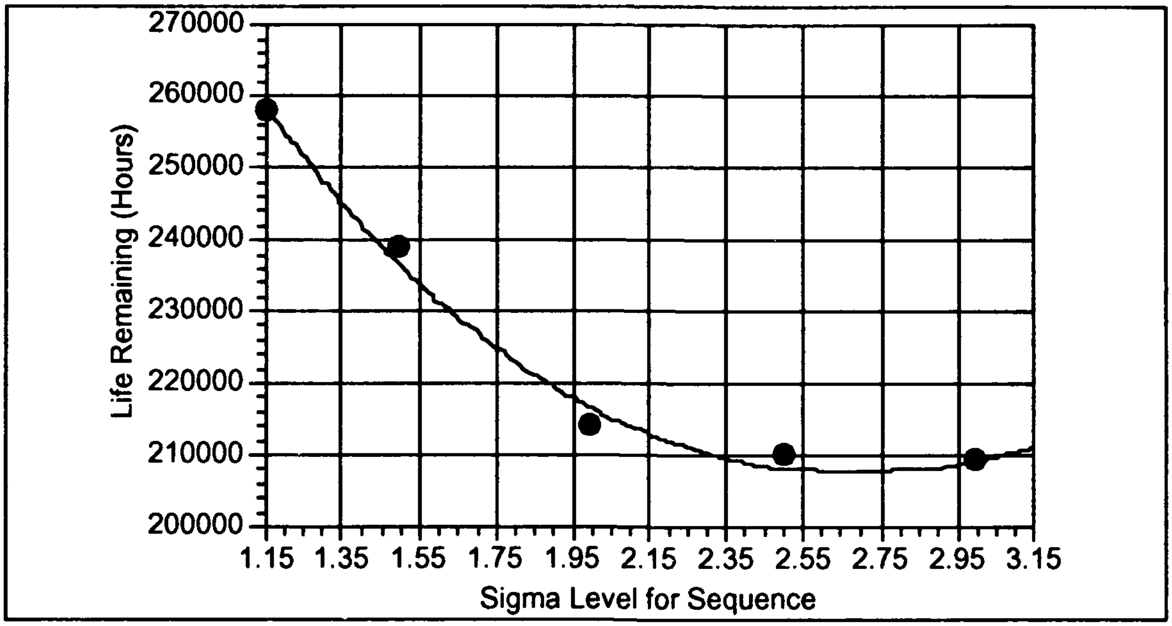

Figure 22: **Life Remaining for Various σ Levels (55 KSI)**

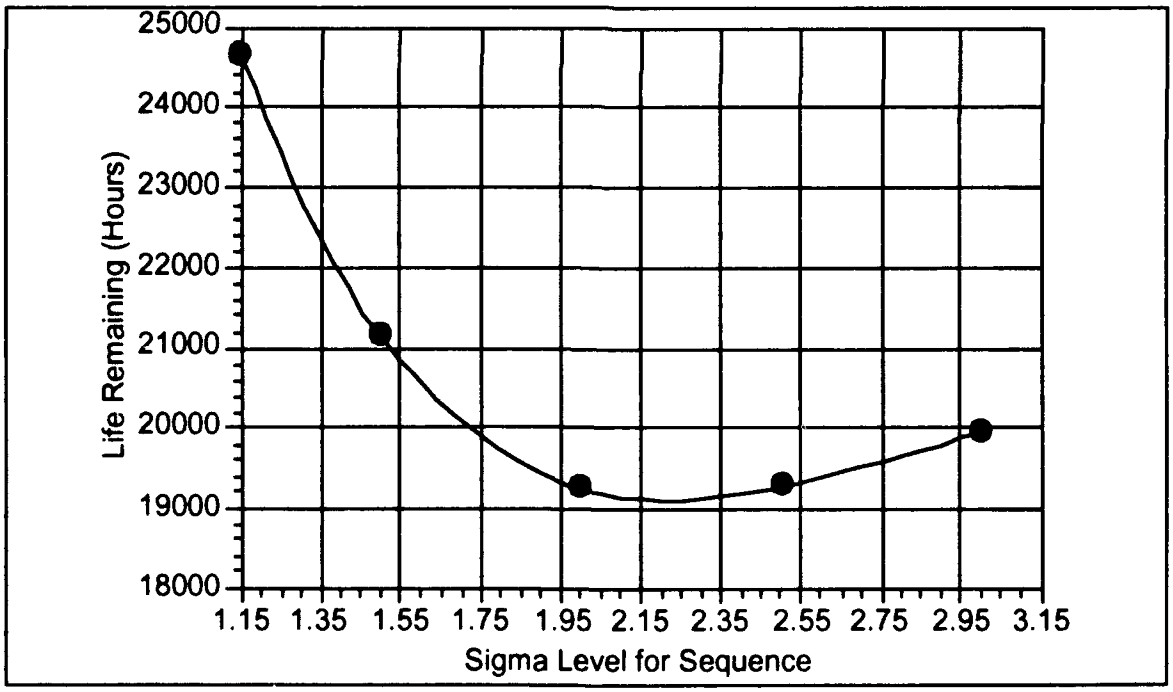

Figure 23: **Life Remaining for Various σ Levels (75 KSI)**

4. Variation of Material Properties

Fatigue calculations were completed for the 1.15s, 1.5s, 2.0s, 2.5s, and 3.0s spectrums while varying the material properties of the metal from 10% to 99.7% as described in section C of this chapter. The runs were made at a reference stress of 55 ksi and 75 ksi. The life remaining in hours from the calculations was presented in Table 17 for the 55 ksi calculations and in Table 18 for the 75 ksi calculations.

TABLE 17: MATERIAL PROPERTIES EFFECT ON LIFE (55 KSI)

MATERIAL PROPERTY	1.15 SIGMA SPECTRUM	1.5 SIGMA SPECTRUM	2.0 SIGMA SPECTRUM	2.5 SIGMA SPECTRUM	3.0 SIGMA SPECTRUM
10% (WEAK)	33630	30413	28398	27202	27118
30%	108522	98734	90576	87433	87530
50%	257837	238611	214046	207906	208969
70%	624789	583575	509789	502854	505919
90%	1872712	1851198	1598996	1535400	1573962
95.50%	4609078	4906298	4445770	4014118	3973645
99.7 %	13557933	15741843	14521858	12330554	12137810

TABLE 18: MATERIAL PROPERTIES EFFECT ON LIFE (75 KSI)

Material Property	1.15 Sigma Spectrum	1.5 Sigma Spectrum	2.0 Sigma Spectrum	2.5 Sigma Spectrum	3.0 Sigma Spectrum
10% (Weak)	6842	5903	5485	5406	5514
30%	14978	12862	11816	11742	12080
50%	24619	21158	19228	19269	19954
70%	39777	34450	30973	31277	32584
90%	69345	61496	54335	55521	58225
95.50%	105982	97328	85338	87744	91491
99.73%	151421	146486	127784	131837	135538

The percentage change in the fatigue life from the average specimen was presented in Table 19 for the 55 ksi calculations and in Table 20 for the 75 ksi calculations. The same information was presented graphically in Figures 24-25. The 50% specimen was considered the average specimen and would possess material properties represented by the average fatigue life of all the specimens. The change in the fatigue life, due to changes in material properties, was significantly greater for the lower reference stress calculations. This was true when varying both the sigma level load sequence and the material properties.

TABLE 19: CHANGE IN LIFE COMPARED TO AVERAGE SPECIMEN (55 KSI)

	10% Material	30% Material	70% Material	90% Material	95.5% Material	99.7% Material
1.15	-86.96%	-57.91%	142.32%	626.32%	1687.60%	5158.34%
1.5	-87.25%	-58.62%	144.57%	675.82%	1956.19%	5582.02%
2	-86.73%	-57.68%	138.17%	647.03%	1977.02%	7254.43%
2.5	-86.92%	-57.95%	141.87%	638.51%	1830.74%	6884.83%
3	-87.02%	-58.11%	142.10%	653.20%	1801.55%	5977.21%

TABLE 20: CHANGE IN LIFE COMPARED TO AVERAGE SPECIMEN (75 KSI)

	10% Material	30% Material	70% Material	90% Material	95.5% Material	99.% Material
1.15	-72.21%	-39.16%	61.57%	181.67%	330.49%	515.06%
1.5	-72.10%	-39.21%	62.82%	190.65%	360.01%	615.67%
2	-71.47%	-38.55%	61.08%	182.58%	343.82%	661.84%
2.5	-71.94%	-39.06%	62.32%	188.14%	355.36%	563.16%
3	-72.37%	-39.46%	63.30%	191.80%	358.51%	560.70%

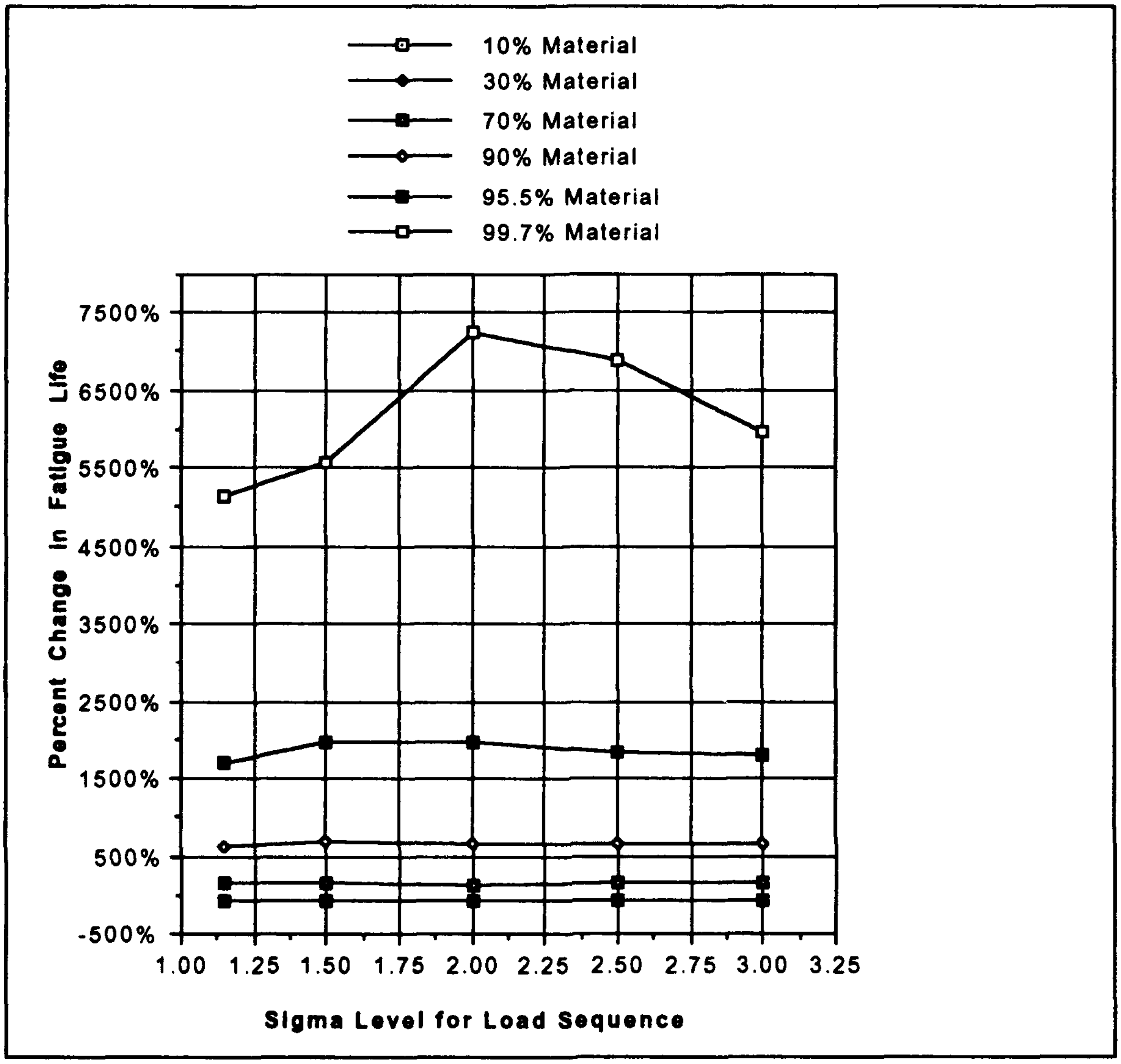

Figure 24: Change in Life Compared to Average Spectrum (55 ksi)

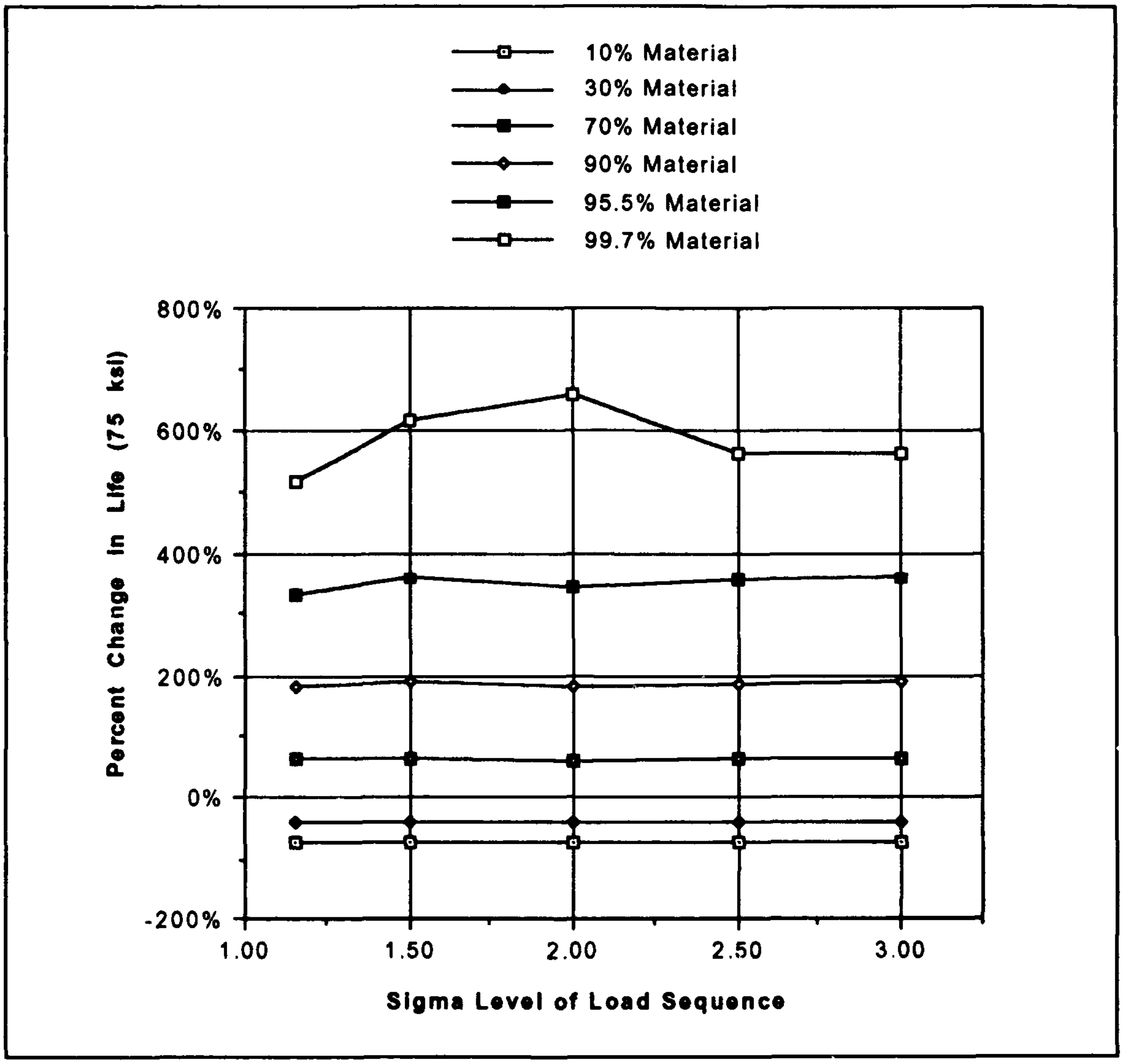

Figure 25: Change in Life Compared to Average Spectrum (75 ksi)

VI. CONCLUSIONS

A. SUMMARY

The results indicated that NAVAIR's requirement for a new aircraft to be designed to withstand a spectrum as severe as 99.73% of the aircraft it is replacing may have little meaning. Significant changes in the fatigue life of the aircraft do not occur between 3.0σ and 2.5σ. The difference between a 3.0σ, 2.5σ, and 2.0σ spectrum was less than 2.7%. Reducing the design requirement to the 2.0σ spectrum would have minimal impact on an aircraft with a design life of 5000 hours or greater. This result was only valid for the A-6 data studied. A different type aircraft, such as the F-14 or FA-18, may have produced in a totally different result.

Calculation of the life while reordering the loads within the spectrum via the randomization method resulted in all life calculations remaining within 9.4% of the mean calculated life. This was in line with a report by Dill and Saff who reported less than a 10% variation in crack growth while reordering loads within a spectrum [Ref. 6].

The variation of life as a function of the NAVAIR reference stress remained linear on a log-log plot from 35 ksi to 100 ksi.

Variation of the material properties of 7075-T651 resulted in approximately a 70% change in the fatigue life for a 10 percentile specimen, while a 30 percentile specimen resulted in a decrease in the fatigue life of approximately 39%.

B. RECOMMENDATIONS

This thesis did not consider the effect that the landings or catapults shots experienced by the aircraft would have on the fatigue life. Landing and catapult data was available in the fatigue spectrum and could have been translated into equivalent loads with the appropriate transfer functions. The distribution of the numbers of landings and catapults per 1000 hours could be analyzed in a similar manner. This would have allowed verification of the accuracy of the current 3σ block spectrum used for the A-6.

Follow on analysis should include the use of A-6 SDRS (Signal Data Recorder Set) data. Computation capabilities are available today that would allow analysis of the large volumes of information produced by the SDRS system. The SDRS data should be screened to provide only actual data on aircraft that are not under flight restrictions.

Finally, further analysis of the data in the tail of each of the g exceedence distributions would confirm whether using a linear regression technique to fit the last 25% of the data points in the unrestricted aircraft database was accurate. Since the data of interest lies in the tail of the distribution, further studies should not spend time trying to model the behavior of the whole population.

Since it is straight forward to obtain a MLE solution for a population with data censored to the right, consideration should be given to fitting the data in the censored population by manipulating the distribution, F, into the complementary function, (1-F). This

would allow censoring to the right and a much simpler solution for the MLE. Appendix B describes the methodology of obtaining the MLE solution for a population that is censored to the right.

APPENDIX A

GENERATION OF AIRCRAFT FATIGUE SPECTRA

A. INTRODUCTION

This appendix introduces a method from the literature used to develop aircraft fatigue spectra. The necessity of an accurate fatigue life spectrum for an aircraft is that it provides a method to predict whether an aircraft will survive its intended lifetime. In the military, the military activity directing an aircraft design requires the manufacturer to provide an expected fatigue spectrum for an aircraft early during the design phase. The USN requires that an aircraft's service life (normally listed in flight hours) be successfully demonstrated (without major failure) to twice the number of spectrum flight hours. In other words, an aircraft with a required service life of 5000 hours would have to successfully undergo a fatigue test utilizing 10,000 spectrum flight hours. The difficult task in spectra generation is predicting the manner in which the aircraft will be flown over its intended lifetime. Valid projections during aircraft introduction may become obsolete if new missions are defined during an aircraft's lifetime, or updates and modifications to the aircraft increase the maximum gross weight or change the center of gravity of the aircraft. Two methods are generally utilized when choosing a fatigue spectrum data base for an aircraft and are discussed below. The chosen data base is then tailored to meet the new aircraft's expected flight use.

1. Standardized Data Bases

The first method utilizes standardized data bases available in the literature, which list normalized loadings that can be expected for a specific type of aircraft. The data bases are generated from aircraft recorded load factor time histories and reduced by an appropriate counting method. To reduce the data to a manageable level, the load cycle data is normally listed in a load matrix. Data bases for transport aircraft fatigue spectra include NASA reports [Ref. 7] and the NLR-Holland and LBF-Germany proposal for standardized loads [Ref. 8]. Data bases for fighter aircraft include the FALSTAFF proposal for standardization [Ref. 9] and MIL-A-87221 [Ref. 10]. Aircraft load data listed in the standardized data bases are normally measured at the center of gravity or measured elsewhere and corrected to the center of gravity. This provides a universal standard that allows data collected from one aircraft to be applied towards a similarly designed aircraft.

2. Similar Aircraft Load Factor Time Histories

The second method of generating a fatigue spectrum utilizes recorded load factor time histories from one aircraft and assumes that the follow on aircraft will undergo the same load history. An example utilizing similar aircraft load factor time histories as a database would be the utilization of a current aircraft's fatigue history to predict the fatigue spectrum of a proposed follow-on aircraft. One would expect this method to provide a more accurate prediction of the fatigue spectrum than the standardized

data bases. Still, many factors will affect the manner in which a new aircraft is flown and will require consideration when generating fatigue spectra. The situation is further complicated with the introduction of digital flight control systems in tactical aircraft. Aircraft with digital flight control systems offer added maneuverability and pilots quickly learn how to utilize the improved capability [Ref. 11]. Thus one might expect a follow on aircraft, designed with digital flight controls and increased maneuverability, to undergo a more severe fatigue spectrum. Additionally, engine developments are allowing the production of aircraft with higher thrust to weight ratios, producing aircraft capable of sustaining higher g loadings for longer periods of time.

B. SPECTRUM DEVELOPMENT

While standardized data bases may provide an accurate load history for a specific type of flight, they often will not accurately predict the spectrum for a new aircraft. A standardized data base may be used to predict the expected load spectrum to be encountered during a specific mission, but the data base may not accurately predict how many of each type of mission will be flown by the new aircraft. An additional problem is continued developments in technology that are producing aircraft that will have a mission profile like no other aircraft. A stealth aircraft that flies only at night will have a much different spectrum than a traditional attack aircraft. Thus, in most cases the designer is left

with the task of modifying the chosen data base to accurately model the actual fatigue spectrum.

Various approaches exist in the literature as techniques to develop a fatigue spectrum. The discussion here is a representative choice to provide the reader with a basic understanding of the techniques used. Consideration of the factors chosen can be varied to meet the degree of complexity that the designer is willing or able to undertake. Spectrum development can be relatively simple or very complex [Ref. 12]. The techniques and considerations considered here for discussion include

1. Creation of the Mission Profile and Mission Segment
2. Environmental Effects
3. Loading Conditions: A Point in the Sky
4. Stress Sequencing

1. Creation of the Mission Profile and Mission Segment

The first step in producing a fatigue spectrum is to define the mission profiles for each type of mission that the aircraft will fly. The mission profiles for a tactical aircraft may include the following missions: air combat maneuvering, high altitude bombing, low altitude bombing, close air support, low level navigation, instrument navigation, maintenance checks, field carrier landing practice, and special weapons delivery. The best prediction of the load sequence for each mission profile can be obtained by using data from a similar type, specially instrumented aircraft. If that

available the next best solution is to define a set of mission segments that can be used to define the mission profile. The mission segments are joined in a segment by segment basis to attain the cycle loading to be expected for a mission profile. Taxi, takeoff, climb, cruise to the warning area, loiter in the training area, return to base, descent, and landing are all mission segments and may be considered and used to obtain the expected spectrum for each mission profile. If the aircraft is a carrier based aircraft, a certain percentage of the takeoffs and landings are considered to be made from the aircraft carrier and an equivalent stress is randomly inserted for the percentage of expected take-offs and landings. Each mission profile is represented by a series of minimum and maximum stress cycles that will be expected in each mission profile.

2. Environmental Effects

Additionally, environmental effects must be considered when generating the mission profiles. Three discussed here include flight control surface position, stresses from store ejection, and ground operation effects on the fatigue spectrum. Again, the three listed here are just a representative sample of what is available in the literature.

The position of the flight control surfaces during maneuvering can significantly affect the stresses imparted on a point on the aircraft. The introduction of digital flight makes predicting the position of any of the flight control surfaces very difficult. Aircraft that have utilized digital flight control computer systems for

control have significantly reduced the ability of traditional methods to accurately calculate the stress that a specific maneuver will generate at a given point on an aircraft [Ref. 11]. The digital flight control algorithms use various parameters to determine the optimum control input to accomplish the desired maneuver commanded by the pilot. For example, a roll at low dynamic pressure may utilize only aileron input while a roll at high dynamic pressure may only use differential stabilator inputs [Ref 13]. Thus each point in the sky results in different wing and control surface loadings which result in large differences in the stress distribution of the aircraft. This proves a difficult problem when trying to calculate the stress loads applied at a point of the aircraft and drives the designer to rely heavily on flight test data to determine actual aircraft stress distributions.

The stresses imparted to the aircraft during store ejection is normally obtained from recorded load factor histories. The equivalent g loads are incorporated into the mission profiles themselves, or a store ejection mission profile can be created and considered when performing the damage calculations. In the latter case, the store ejection mission profile is randomly inserted into the fatigue calculations in the same manner in which a mission profile would be inserted. MIL-A-8868B requires consideration for store release up to 6.0 g's of acceleration plus the associated g jump that occurs during store release. Ground operation effects on load spectra normally consider taxi, braking, turning, take-off roll, and landing

roll out. Aircraft towing, stores loaded on the aircraft, and the effects of a pitching aircraft carrier on aircraft tied to the flight deck may also need to be considered. The FALSTAFF method utilized fixed taxi-load sequences for each flight, using 2 full cycles before and after each flight to represent the load levels applied to the aircraft. The military requirements listed in MIL-A 8866(AS) MIL-A-8868(AS) give specific details that define the environment to be expected during each ground taxi evolution, including types of breaking on landing rollout, effects of antiskid devices, pivoting during taxi, vertical gear inputs during taxi, and effect of engine run-up. To illustrate the specificity of the design requirements, the Navy's definition of braking during ground maneuvers follows: "Hard braking with maximum braking effect (0.8 coefficient of friction) shall occur twice per taxi run and medium braking with half maximum effect shall occur an additional five times per run. Turning with total side loads of 0.4 times the airplane weight applied as inboard and alternately as outboard loads shall occur five times per taxi run. Pivoting with 1/2 limit torque load shall occur once per three taxi runs." [Ref. 14] Accurate predictions of the stresses imparted to the aircraft structure by ground operations are very difficult to predict thus, instrumented aircraft may be the best source of accurately predicting ground maneuver loadings.

3. Loading Conditions: A Point in the Sky

The loading conditions at which each particular fatigue cycle in the fatigue spectrum is applied must be defined for each

mission profile. The designer must take into consideration various parameters at the time of load application, including aircraft airspeed, altitude, gross weight, configuration, center of gravity, and point on the aircraft being analyzed. While the most accurate representation of the fatigue life would occur by relating every single load factor occurrence to the specific parameters under which it occurred, this is not reasonable or possible. Predicting several hundred thousand load occurrences over an aircraft's lifetime is unreasonable. Thus flight parameters are broken into segments and averaged to reduce the level of complexity [Ref. 12]. The choosing of representative loading conditions must be based on knowledge of how the aircraft is to be flown. A study of the aircraft's mission profiles and aircraft limitations should allow the designer to accurately predict the loading conditions of the aircraft. For all missions the designer might know that 30% of the aircraft's fuel will be burned traveling to a training area in peace time or to the engagement area in wartime. For fighter missions, the aircraft may take off at a different gross weight or in a different configuration than a bombing mission. Strafing runs may be completed after the dropping of external stores from the aircraft. An aircraft limitation might prevent maximum maneuvering until all external fuel tanks are empty of fuel. Carrier landings normally require greater fuel reserves onboard at landing, so carrier landings may occur at higher gross weights than airport landings. The designer may use this type

of information to weight sections of each mission segment before they are compiled into the mission profiles.

A "point in the sky" is normally defined for each mission profile or could even be defined for each mission segment if the resources and knowledge allow. The "point in the sky" chosen is considered to be representative of the conditions that the aircraft will encounter while maneuvering during a mission profile. The "point in the sky" defined should include information about the aircraft's airspeed, altitude, gross weight, and configuration.

4. Stress Sequencing

The next task requires arranging the stresses into a stress spectrum. Normally it is not possible to predict the order of the mission profiles or the orders of the loads in each individual mission profile. Therefore a random order is normally utilized to order the load sequence. The mission profiles are chosen in a random order and the loads within each mission segment are drawn in a random order.

Load matrices for an entire aircraft sequence are very large and contain hundreds or thousands of numbers. The large numbers of cycles in a fatigue spectrum are normally stored in a loads matrix as shown in Figure 26 [Ref. 9]. To illustrate, in Figure 26, the load would vary from level 4 to level 1, four times during the load sequence.

Finally damage calculations are performed by pulling sequences from the load matrix on a random bases. The computer

algorithm must be written to ensure that all loads are drawn only one time an all cycle sequences can be completed.

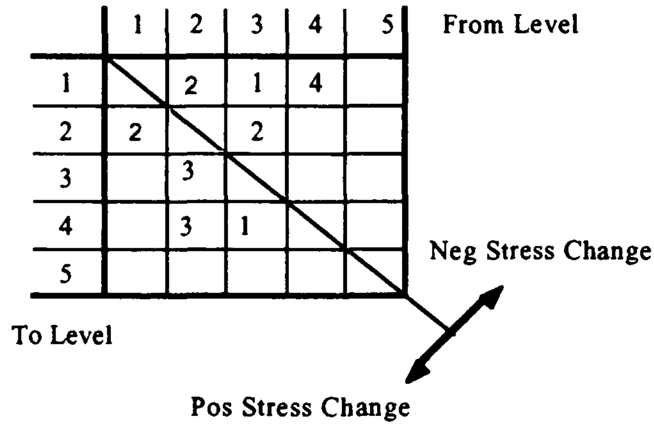

Figure 26: Loads Matrix

APPENDIX B

MLE Method for Censored Data

Maximum likelihood estimation (MLE) are utilized for estimating population parameters. This appendix examines the utilization of the MLE method to provide the shape parameter, α, and the scale parameters, β, for the two parameter Weibull distribution with data censored to the right.

The Weibull CDF and PDF are described by Equations 1 and 2.

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right) \tag{9}$$

$$f(x) = \alpha * \frac{x_i^{(\alpha-1)}}{\beta^{\alpha}} * \exp\left\{-\left(\frac{x_i}{\beta}\right)^{\alpha}\right\} \tag{10}$$

$$F(x) = \int f(x)dx \tag{11}$$

The likelihood, L, is defined as the product of the density function (PDF) evaluated at each data point. If the values for the parameters, α and β, can be found that maximize the likelihood, then we have maximized the chance that the PDF and CDF will describe the data set. The solution of the two equations obtained by taking the partial differential of L with respect to α and β, and setting both differentials equal to zero, will yield the maximum likelihood values for the parameters. L(x;α) is read the likelihood of x given α, and

$L(x;\beta)$ is read the likelihood of x given β. We are interested in maximizing $L(x;\alpha,\beta)$ Derivation of the solution follows:

$x_1, x_2, x_3, \ldots x_m$ Data of interest

$x_{m+1}, x_{m+2}, \ldots x_n$ Censored Data

$x_{m+1} > x_{m+2} > x_{m+3} \ldots$

$$f = f(\alpha,\beta) \tag{12}$$

The likelihood function is:

$$L = f(x_1,\alpha,\beta) * f(x_2,\alpha,\beta) * f(x_3,\alpha,\beta) * \cdots f(x_n,\alpha,\beta) \tag{13}$$

or

$$L = \left[\prod_1^m f(x_i)\right] * \left[\prod_m^n \left\{\int_{x_m}^{\infty} f(x)dx\right\}\right] \tag{14}$$

Substituting in F(x) as per equation 3.

$$L = \left[\prod_1^m f(x_i)\right] * \left[\prod_{m+1}^n \{1 - F(x)\}\right] \tag{15}$$

$$\prod_1^n [1 - F(x)] = [1 - F(x_m)]^{(n-m)} \tag{16}$$

Substituting Equation 8 in Equation 7

$$L = \left[\prod_1^m f(x_i)\right] * [1 - F(x_m)]^{(n-m)} \tag{17}$$

Substituting Equation 1 and 2 for F(x) and f(x) of Equation 9..

$$L = \left[\prod_{1}^{m} \alpha * x_i^{(\alpha-1)} * \beta^{-\alpha} * \exp\left\{-\left(\frac{x_i}{\beta}\right)^{\alpha}\right\}\right] * \left[\exp\left\{-\left(\frac{x_m}{\beta}\right)^{\alpha}\right\}\right]^{(n-m)} \tag{18}$$

To remove the products take the natural logarithm of L.

$$\ln(L) = m\ln(\alpha) - m\alpha\ln(\beta) + \sum_{1}^{m}(\alpha-1)\ln(x_i) - \beta^{-\alpha}\sum_{i}^{m} x_i^{\alpha} - (n-m)\beta^{-\alpha}x_m^{\alpha} \tag{19}$$

Differentiating Equation 8 w.r.t. α and setting to zero.

$$\begin{aligned} 0 = m\alpha^{-1} - m\ln\beta + \sum_{1}^{m}\ln x_i + \beta^{-\alpha}\sum_{i}^{m}\left[x_i^{\alpha}\ln(x_i)\right] + \beta^{-\alpha}\ln\beta\sum_{1}^{m}x_1^{i} - \\ (n-m)\beta^{-\alpha}x_m^{\alpha}\ln(x_m) + (n-m)\beta^{-\alpha}\ln(\beta)x_m^{\alpha} \end{aligned} \tag{20}$$

Differentiating Equation 8 w.r.t. β and setting to zero.

$$0 = -m\alpha\frac{1}{\beta} - \left[(-\alpha)\beta^{(-\alpha-1)}\sum_{1}^{m}x_i^{\alpha}\right] - \left[(n-m)(-\alpha)\beta^{(-\alpha-1)}x_m^{\alpha}\right] \tag{21}$$

Solving Equation 10 for β.

$$\beta = \left\{m^{-1}\left[\sum_{1}^{m}x_1^{\alpha} - (n-m)x_m^{\alpha}\right]\right\}^{\frac{1}{\alpha}} \tag{22}$$

Finally, substituting Equation 11 into Equation 9 and Solving for α.

$$\alpha = \left\{ \frac{\left[\frac{1}{m} \sum_{1}^{m} \left\langle x_i^{\alpha} \ln(x_i) \right\rangle + \frac{(n-m)}{m} x_m^{\alpha} \ln x_m \right]}{\frac{1}{m} \sum_{1}^{m} x_i^{\alpha} + \frac{(n-m)}{m} x_m^{\alpha}} - \frac{1}{m} \sum_{1}^{m} \ln(x_i) \right\}^{-1} \tag{23}$$

We note that for censoring to the left, Equation 13 through 17 have to be modified.

$$L = \left[\prod_{1}^{m} f(x_i) \right] * \left[\prod_{m}^{n} \left\{ \int_{-\infty}^{x_m} f(x) dx \right\} \right] \tag{14'}$$

Leading to:

$$L = \left[\prod_{1}^{m} \alpha * x_i^{(\alpha-1)} * \beta^{-\alpha} * \exp\left\{ -\left(\frac{x_i}{\beta} \right)^{\alpha} \right\} \right] * \left[1 - \exp\left\{ -\left(\frac{x_m}{\beta} \right)^{\alpha} \right\} \right]^{(n-m)} \tag{18'}$$

From this point on the process of taking the logarithm and the calculus of maximization can no longer be carried out explicitly. Numerical solutions to maximize L must be used.

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